UNIT 14 MEASURES OF CENTRAL TENDENCY

Structure

14.1 INTRODUCTION

As a teacher, we always deal with the data gathered from assessment of student's performance in continuous as well as terminal examinations. Many a time, we get engaged with analyzing the data/score of the students and compare them with other students or groups. We also prepare the progress report of the students in which we use certain statistics to describe the performance of the students in a meaningful and understandable way. We analyze the individual score of the students as well as the group performance of the sections, classes and schools. We also compare the performance of students between sections, classes and schools.

Keeping in view the above, the present Unit discusses the concept, process and methods of calculation of measures of central tendency of the scores. In the previous Unit, you have studied about the nature of data. In this Unit, you will recapitulate the concept of individual and the group data. Further the Unit will make you acquainted with various methods used for calculation of measures of central tendency i.e. Mean, Median and Mode and also make you understand the interpretation and use of the Mean, Median, and Mode. The Unit also makes you acquainted with establishing relationships and comparisons among Mean, Median and Mode.

14.2 OBJECTIVES

After going through this Unit, you will be able to:

- differentiate between individual and group data;
- explain the meaning of measures of central tendency;
- compute mean from ungrouped and grouped data and interpret it;
- compute and interpret the combined mean;
- compute median from ungrouped and grouped data and interpret it;
- compute mode from ungrouped and grouped data and interpret the it;
- discuss the uses and limitations of mean, median and mode;
- discuss the relationship of mean, median and mode;
- compare the mean, median and mode for their relative importance in a given context; and
- select an appropriate measure of central tendency as per the nature of data and purpose.

14.3 INDIVIDUAL AND GROUP DATA

In the previous Unit, you have studied the nature of data, its organization, making frequency distribution table and converting raw scores into group scores. To recapitulate what you have studied earlier, we can say that the raw or the individual score keeps very little meaning because it lacks value of interpreting the score with other scores. At the same time, without knowing other individual scores it is difficult to analyze the order and the position of an individual score in the group. For example, let Sohan scored 50 in a test of Mathematics. What does the score mean? How many students are below or above the score of Sohan? Whether Sohan is an average, below average or above average student? These questions can't be answered simply from the individual score. To make the individual score more meaningful, we can convert it into group scores. Group scores can be presented in terms of average score, equal intervals, and also ordering it both in ascending and descending order. Individual scores can be presented as group score in terms of frequency distribution table. The individual (raw score) and the group score have been presented in the following table.

Table 14.1: Individual/Raw score and the group score

Individual/Raw Score			Group Score	
Name	**Identification**	**Score**	**Class Interval**	**Frequencies**
Sohan	X_1	45	55 - 59	1
Ritu	X_2	20		
Zakir	X_3	11	50 - 54	2
Suresh	X_4	33		
Sania	X_5	53	45 - 49	2
Manju	X_6	57		
Rehan	X_7	46	40 - 44	2
Kanak	X_8	24		
Roni	X_9	29	35 - 39	2
Deekshya	X_{10}	30		
Kranti	X_{11}	19	30 - 34	3
Samir	X_{12}	36		
Subrat	X_{13}	42	25 - 29	1
Manjit	X_{14}	38		
Abdul	X_{15}	21	20 - 24	4
John	X_{16}	50		
Karan	X_{17}	17	15 - 19	2
Harihar	X_{18}	23		
Mamta	X_{19}	43	10 - 14	1
Deep	X_{20}	33		
	N = 20			**N = 20**

The above table shows the individual score which can also be called as raw score and it is converted to group score which is presented in Class Intervals with their frequencies. Getting raw score is the first step to go forward and interpreting it in different meaningful ways, mean, median, or mode of the scores. The mean, median and the mode can be calculated in various methods depending upon the nature of the data. In the next section, we will discuss the concept of Measures of Central tendency and how to calculate the central tendency.

Activity 1

Referring table 14.1, convert the following individual/raw scores into group scores by representing it in class intervals and frequencies.

Individual/raw Scores:

X_1 – 20; X_2 – 18; X_3 – 52; X_4 – 45; X_5 – 37; X_6 – 38; X_7 – 24; X_8 – 17; X_9 – 48; X_{10} – 40; X_{11} – 50; X_{12} – 52; X_{13} – 33; X_{14} – 26; X_{15} – 31; X_{16} – 14; X_{17} – 44; X_{18} – 29; X_{19} – 56; X_{20} – 63; X_{21} – 67; X_{22} – 55; X_{23} – 30; X_{24} – 27; X_{25} – 35; X_{26} – 41; X_{27} – 31; X_{28} – 19; X_{29} – 54; X_{30} – 22 (N = 30)

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14.4 MEASURES OF CENTRAL TENDENCY

As a teacher, you might have engaged in analyzing variety of data relating to school as well as performance of the students in different subjects. Again, you might have come across to analyse the data for preparing the report card and progress of the students. In such cases, quite a time, we need to use certain statistical techniques to make the individual score meaningful by comparing it with the group scores. The most common statistics that we use in school is the measures of central tendency. Central tendency is the central value or score of a group. For example, let the score of 30 students in a particular subject is given, we can calculate the average score of that group and then we can compare the individual score with the central value/score of that group. Again, central tendency also helps us to compare the central values of two different groups it may be within different sections of a same class, between two different classes in the same school and also between same standards in two different schools. We use measures of central tendency for making inter and intra group comparisons. The most commonly used measures of central tendency are:

- Mean
- Median
- Mode

14.4.1 Scales of Measurement and Measures of Central Tendency

In Unit-13, you have studied the scales of measurement. We usually use four types or scales of measurement, i.e. nominal, ordinal, interval and ratio scale. In educational measurement, generally, we use nominal, ordinal and equal interval scale. Ratio scale is used in measurement of physical sciences where there is a concept of absolute zero point in measurement. But in educational measurement, we consider zero point measurement as the relative zero point. The use of different methods of calculating measures of central tendency depends upon the nature of the data and its scales of measurement.

Data on Nominal Scale: If data is on nominal scale which is mostly qualitative in nature, we can simply count the number of cases in each category and obtained frequencies. In such cases, if the data is in nominal scale, we can use the statistics 'The Mode' as the measure of central tendency. It supports the definition of the mode, i.e., 'mode is the most frequent item/score of the group'.

Data on Ordinal Scale: Ordinal scale implies that the scores are in an order, either ascending or descending. When data is arranged in rank order the measure of central tendency found by locating a point that divides the whole distribution into two equal halves. In such cases, we use the statistics 'median' as the measure of central tendency. It supports the definition of median, i.e., 'median is a point on the scale of measurement below and above which lie exactly 50 percent of the cases. It may be noted that median is defined as a point and not as a score or any particular measurement.

Data on Equal Interval Scale: When data is presented in class intervals and there is scope to convert the data into class intervals, we can best use statistics 'Mean' as the measure of central tendency. Mean is the most popular measures

of central tendency which is highly used in schools. Mean of a distribution of scores may be defined as the point on the scale of measurement obtained by dividing the sum of all the scores by the number of scores. Mean is also called as the most appropriate measures of central tendency.

In the next section, we will discuss the methods of calculating Mean, Median and Mode as the measures of central tendency.

14.5 THE MEAN

As discussed above, mean is calculated when the data is presented or have the scope to present it in interval scale. It is also popularly known as 'Arithmetic Mean'. Mean can be calculated by adding sum of the observations/scores by dividing by total number of cases. The formula of calculating Mean is as follows:

$$Mean\ (M) = \frac{\sum X}{N}$$

Where, $\sum X$ = Sum of all the scores/values

N = Total number of cases

Mean can be calculated both in ungrouped data as well as grouped data.

14.5.1 Calculation of Mean for Ungrouped Data

When only raw data is given, Mean can be calculated by adding all the raw scores and dividing it by the total number of the raw scores.

Example: The scores of ten students in a class is given as follows. Calculate the Mean of the scores.

Scores: 15, 37, 22, 11, 40, 29, 32, 45, 20, and 30. (N = 10)

$$Mean\ (M) = \frac{\sum X}{N}$$

['X' is the individual score and 'N' is total number of cases]

= (15+37+22+11+40+29+32+45+20+30) / 10

= 281 / 10

= 28.1 (Answer)

14.5.2 Calculation of Mean for Grouped Data

We can calculate Mean in three situations, i.e.

1. When the scores and the frequencies are given
2. When data is arranged in frequency distribution table i.e. Class Intervals as well as Frequencies are given.
3. When data is arranged in frequency distribution table i.e. Class Intervals as well as Frequencies are given. (by using Assumed Mean Method is also called as short method)

1. Calculation of Mean when Scores and Frequencies are given

We can calculate Mean by using a simple formula when scores and frequencies are given. Let us solve with an example.

Example: Calculate mean of the following data:

Score	25	31	33	42	46	51	55	58	60	72
Frequencies	3	7	9	12	13	6	4	3	2	1

Solution:

$$Mean\ (M) = \frac{\sum fX}{N}$$

?fX = Summation of frequencies multiplied with the scores

N = Total frequencies

Score (X)	Frequencies (f)	fX
25	3	75
31	7	217
33	9	297
42	12	504
46	13	598
51	6	306
55	4	220
58	3	174
60	2	120
72	1	72
	N = 60	**$\sum fX$ = 2583**

$$Mean\ (M) = \frac{\sum fX}{N}$$

$$= 2583 / 60 = 43.05 \text{ (Answer)}$$

2. Calculation of Mean when Data are given in Class Intervals with Frequencies

In group data, when the class intervals as well as frequencies are given, we can calculate Mean by using the same formula. This method is also called as long method for calculating Mean. The formula for calculating Mean is as follows:

$$Mean\ (M) = \frac{\sum fX}{N}$$

X = Mid-point of the Class Interval

f = Frequencies

N = Total number of cases ($\sum f$)

Example : Compute the Mean for the following frequency distribution.

Class Interval	40-44	35-39	30-34	25-29	20-24	15-19	10-14
Frequencies	3	5	10	14	8	6	4

Solution :

Score (X)	Frequencies (f)	Mid Point (X)	fX
40-44	3	42	126
35-39	5	37	185
30-34	10	32	320
25-29	14	27	378
20-24	8	22	176
15-19	6	17	102
10-14	4	12	48
	N = 50		**$\sum$fX = 1335**

$$Mean\ (M) = \frac{\sum fX}{N}$$

= 1335 / 50

= 26.7 (Answer)

3. Calculating Mean by using Assumed Mean Method

In group data, when the data is presented in class intervals with frequencies, we can calculate Mean by using long method as well as assumed mean or short method. Assumed mean method is widely used to calculate mean in this situation as because to avoid lengthy calculations of multiplications of mid-points of class intervals with their corresponding frequencies. In assumed mean method, we assume a class, assuming that the mean lies in that class. We follow certain steps to calculate Mean in assumed mean method. The following formula is used to calculate Mean in assumed mean method:

$$Mean\ (M) = A.M. + \frac{\sum fd}{N} \times c.i.$$

A.M. = Assumed Mean

f = Frequencies

d = Deviation from assumed mean

N = Total number of frequencies

c.i. = Size of the Class Interval

Steps followed for calculating Mean in Assumed Mean Method:

Step 1 : Calculation of assumed mean

(Assumed mean is generally the mid-point of the class having highest frequency).

Step 2 : Calculate the mid-point of the Class

[$Mid\ point = \frac{Lower\ Limit\ of\ the\ Class + Higher\ Limit\ of\ the\ Class}{2}$]

Step 3 : Calculation of deviation (d)

[Deviation (d) can be calculated by using the method like : $d = \frac{Mid\ point\ of\ the\ Class - A.M.}{Size\ of\ the\ Class}$

Step 4 : Find out multiplications of frequency and corresponding deviation and place the obtained value in the column headed by *fd*.

Step 5 : Find the sum of the column, i.e. $\sum fd$

Step 6 : Apply the formula

Let us calculate mean by using Assumed Mean method by an example:

Example: Find out Mean of the distribution by using Assumed Mean method.

Class Interval	50-54	45-49	40-44	35-39	30-34	25-29	20-24	15-19	10-14
Frequencies	2	3	5	7	10	6	4	2	1

Solution :

Score (X)	**Mid Point (X)**	**Frequencies (f)**	**Deviation (d)**	**fd**
50-54	52	2	+4	8
45-49	47	3	+3	9
40-44	42	5	+2	10
35-39	37	7	+1	7
30-34	32 (A.M.)	10	0	0
25-29	27	6	-1	-6
20-24	22	4	-2	-8
15-19	17	2	-3	-6
10-14	12	1	-4	-4
		N = 40		**$\sum fd$ = 10**

For calculating the values, the above mentioned steps are used.

$$Mean\ (M) = A.M. + \frac{\sum fd}{N} \times c.i.$$

A.M. = 32

N = 40

c.i. = 5

$\sum fd$ = 10

$$Mean\ (M) = 32 + \frac{10}{40} \times 5$$

= 32 + 1.25

= 33.25 (Answer)

Note: *After solving such questions you will see that you always get values +1, +2, +3 etc. on one side and -1, -2, -3 etc. on the other side of the class interval chosen for Assumed Mean. In fact it becomes just a mechanical process, after some time, to find deviations i.e. first putting Zero (0) against the column of Assumed Mean and putting +1, +2, +3 towards class intervals with bigger score limits and -1, -2, -3 towards class intervals shown with smaller score limits. This may help you save time as well. (B.Ed. ES-333, IGNOU, 2010)*

14.5.3 Calculation of Combined Mean

As like calculation of Mean is very much important for the part of a teacher, accordingly calculation of Combined Mean is also equally important. Quite a time, you might have come across the situation that in a particular class, there may be many sections and you have to analyse their scores in different subjects. The number of students in each sections are also sometime different. In such cases, you can calculate a combined mean of the entire groups if the data relating to mean score and the Number of students in each group is given. Similarly if we have the means for various schools and the district mean is required, it would also call for computing the combined mean. Combined Mean can be calculated by using the following formula:

$$Combined\ Mean\ (M.comb.) = \frac{N1M1 + N2M2 + N3M3 + ---}{N1 + N2 + N3 + ---}$$

N1 = Number of the first group

N2 = Number of the second group

N3 = Number of the third group

M1 = Mean of the first group

M2 = Mean of the second group

M3 = Mean of the third group

Let us calculate mean by using the above formula.

Example : There are three sections in Class-IX of a school, section A, B and C. The Mean score in Mathematics of each sections with the number of students are given as follows. Find the combined mean of the students in Mathematics.

Sections	**A**	**B**	**C**
Mean Scores	**82**	**65**	**70**
Number of Students	36	33	41

Solution:

$$Combined\ Mean\ (M\ comb.) = \frac{N1M1 + N2M2 + N3M3}{N1 + N2 + N3}$$

[N1 = 36; N2 = 33; and N3 = 41]

[M1 = 82; M2 = 65; and M3 = 70]

$$Combined\ Mean\ (M\ comb.) = \frac{(36 \times 82) + (33 \times 65) + (41 \times 70)}{36 + 33 + 41}$$

$$= \frac{2952 + 2145 + 2870}{110}$$

$$= \frac{7967}{110}$$

= 72.43

From the above example, you may notice that the combined mean is 72.43, where as the individual mean of different sections are 82 in section A, 65 in section B and 70 in section C. Now you can compare between average performance of students in a section with the combined mean of all the sections. If someone erroneously adds the mean scores of all the sections and divide it by the number of sections, the average will come as 72.33, which is not same as the calculated combined mean. It is therefore, for getting the multi group average, better to calculate their combine mean by using the appropriate formula, not simply by calculating the average.

14.5.4 Use of Mean

Mean is called as the most appropriate measure of central tendency as it considers the value of each and every score of the group for calculating the mean. Mean is referred because of its high reliability and its applicability to inferential statistics. Mean provides a clear idea about how the scores are varied from the central value. Mean is used when:

- The data is distributed symmetrically, i.e. distributions are not marked skewed.
- We wish to know the centre of gravity of a sample.
- Central tendency with greater stability is wanted.
- Other statistics (standard deviations, coefficient of correlation etc.) for inferential purposes are to be calculated.
- Group performances are to be calculated with accuracy and precision.
- Comparing the intra and the inter group students.

14.5.5 Limitations of Mean

Besides the use of Mean it has also its own limitations. The limitations can be listed as follows:

- A single extreme score (may be lowest or highest) may influence the value of mean. As example, let the daily wages of five persons are Rs. 100.00; Rs.150.00; Rs. 170.00; Rs.160.00 and Rs. 3000.00. The average daily wages comes as Rs.716.00. In this case, the only one extreme score influence the group average. A person who is getting Rs.100.00 per day, is calculated as Rs. 716.00 as average. This is not justified.
- There are situations, where mean may not provide meaningful information.
- Works only when all values are equally important.

Check Your Progress 1

Note: a) Write your answers in the space given below.

b) Compare your answer with the one given at the end of the Unit.

[Put ( ) mark in appropriate answer]

1. Mean is used when data is presented in :

   a) Nominal Scale b) Ordinal Scale c) Equal Interval Scale

2. Mean is the appropriate measure of central tendency when the data is :

   a) Symmetrical b) Positively Skewed c) Negatively Skewed

3. Mean is the most reliable method of finding out the central tendency, because :

   a) It is easy to calculate

   b) Each score is given weightage to calculate mean

   c) Can calculate mean by assumed mean method

   d) It is highly used in research

4. Combined Mean is calculated, when :

   a) Comparison is needed among the groups

   b) Multi group comparison is needed in case the numbers of students in the groups are unequal.

   c) Comparison of mean scores is needed between two or more schools.

   d) All of the above.

14.6 THE MEDIAN

You have studied in the section 14.4.1 of this Unit that Median is used when the scores are presented in ordinal scale, i.e. possible to organize either in descending order or ascending order. Median is defined as it is the 'mid point of the series below and above which lie exactly 50 percent of the cases'. For calculating Mean, scores are given importance whereas for calculating Median, number of items are given importance. It may be noted that median is defined as a point on the series and not as a score or any particular measurement. The score is identified indirectly calculating from the numbers. As like Mean, Median can also be calculated by using both ungrouped and grouped data.

14.6.1 Calculation of Median for Ungrouped Data

For calculating Median in ungrouped data, we use a very simple formula i.e. (N+1)/2 th item in the series in order of ascending or descending order. There are certain changes in calculating Median, when the number of items in the series is in odd or even numbers.

Median in Ungrouped Data (When number of items are odd)

We use the following formula for calculating Median when the total number of items in the group are odd:

$$Median = \frac{N+1}{2} \; th \; item \; in \; the \; series$$

[Where, N is total number of cases]

Let us calculate it with an example:

Example : Find the Median of the following scores:

Scores : 9, 22, 32, 19, 12, 26, 15 (N = 7)

Solution :

Step 1 : Arrange the scores either in ascending or descending order (Let us arrange it in ascending order).

Step 2 : Use the formula for calculating Median

Step 3 : Determine the Median score from the item

Scores (in ascending order) : 9, 12, 15, 19, 22, 26, 32

N = 7

$$Median = \frac{N+1}{2} \; th \; item \; in \; the \; series$$

$$= \frac{7+1}{2} \; th \; item$$

$$= \frac{8}{2} \; th \; item$$

= 4^{th} item

= 19 (Answer)

4^{th} item of the series in ascending or descending order is 19.

Median for Ungrouped Data (When number of items are even)

We use the following formula for calculating Median when the total number of items in the group are even:

$$Median = \frac{N+1}{2} \; th \; item \; in \; the \; series$$

[Where, N is total number of cases]

Let us calculate it with an example:

Example : Find the Median of the following scores:

Scores : 9, 22, 32, 19, 12, 26, 15, 25 (N = 8)

Solution :

Step 1 : Arrange the scores either ascending or descending order (Let us arrange it in ascending order).

Step 2 : Use the formula for calculating Median

Step 3 : Determine the Median score from the item

Scores (in ascending order) : 9, 12, 15, 19, 22, 25, 26, 32

N = 8

$$Median = \frac{N+1}{2} \; th \; item \; in \; the \; series$$

$$= \frac{8+1}{2} \; th \; item$$

$$= \frac{9}{2} \; th \; item$$

= 4.5^{th} item

$$= \frac{4th \; Item + 5th \; Item}{2}$$

$$= \frac{19+22}{2}$$ *[**Note**: 4^{th} item is 19 and 5^{th} item is 22]*

$$= \frac{41}{2}$$

= 20.5 (Answer)

4.5^{th} item of the series is 20.5

14.6.2 Calculation of Median for Grouped Data

You have studied that the Median is the mid point of the series in order below and above which 50 percent of the cases lie. For calculating Median in grouped data, the assumption made is that frequencies are evenly distributed within the class interval. The following formula we use for calculating Median in grouped data:

$$Median = L + \frac{\frac{N}{2} - cfb}{fm} \times \text{c. i.}$$

Where :

L = Lower limit of the median class

N = Total number of the cases

cfb = Cumulative Frequency below the median class

fm = Frequency of the median class

c.i. = Size of the Class Interval

Steps followed for calculation of Median :

Step 1 : Find out the Median Class (the class which follows N/2th value; can be seen from the cumulative frequency).

Step 2 : Calculate Lower Limit of the Median Class by subtracting 0.5 from the Lower Limit of the Median Class.

Step 3 : Find out cfb; is the cumulative frequency below the Median Class.

Step 4 : Find out fm; is the exact frequency of the Median Class.

Step 5 : Find out the size of the class

Step 6 : Apply the formula to calculate Median

Example : Calculate median for the following frequency distribution

Class Interval	90-94	85-89	80-84	75-79	70-74	65-69	60-64	55-59	50-54	45-49
Frequencies	2	3	3	4	7	12	9	4	2	2

Solution :

Class Interval (C.I.)	**Frequencies (f)**	**Cumulative Frequency (cf)**
90-94	2	48
85-89	3	46
80-84	3	43
75-79	4	40
70-74	7	36
65-69 (Median Class)	**12 (fm)**	**29**
60-64	9	**17 (cfb)**
55-59	4	8
50-54	2	4
45-49	2	2
	N = 48	

$$Median = L + \frac{\frac{N}{2} - cfb}{fm} \times \text{c.i.}$$

N/2 = 24

L = 64.5

cfb = 17

fm = 12

c.i. = 5

$$Median = 64.5 + \frac{24 - 17}{12} \times 5$$

$$= 64.5 + \frac{7}{12} \times 5$$

$$= 64.5 + \frac{35}{12}$$

$$= 64.5 + 2.92$$

= 67.42 (Answer)

Special Case for Calculation of Median

There may arise a special situation where there are no cases within the interval containing the Median. Let us take an example.

Example : Find the Median for the following frequency distribution :

Class Interval	26-28	23-25	20-22	17-19	14-16	11-13	8-10	5-7
Frequencies	2	2	7	6	0	9	7	1

Solution :

Class Interval (C.I.)	**Frequencies (f)**	**Cumulative Frequency (cf)**
26-28	2	34
23-25	2	32
20-22	7	30
17-19	6	23
14-16 (Median Class)	**0**	**17**
11-13	9	17
8-10	7	8
5-7	1	1
	N = 34	

If you examine the above table for computing median you come across $\frac{N}{2}$ = $\frac{34}{2}$ = 17 against two Class Intervals showing cumulative frequencies. If one calculates Median mechanically, there is the possibility to arrive at a wrong answer. So, it will not be so simple to apply the formula and get the results. Also, if you calculate cumulative frequency from above, another class interval (17-19) may also show 17 as cumulative frequency. We will solve this question in two alternative ways (i) conceptually and (ii) empirically.

Solution 1 :

On counting number of cases from top and bottom sides we find that there are 17 cases above 16.7 and 17 cases below 13.5. We come across two points (instead of one) below and above which lie exactly fifty percent cases as the class interval 14-16 is void (without any frequency). So extending the assumption to this class, (that frequencies are evenly distributed within each class

interval). We may add half of this void class interval to either side and find median.

So Median = 13.5 + 3/2 = 13.5 + 1.5 = 15 (Answer)

Or Median = 16.5 – 3/2 = 16.5 – 1.5 = 15 (Answer)

Solution 2 :

C.I.	f	c.f.	Modified C.I.	f	c.f.
26-28	2	34	26-28	2	34
23-25	2	32	23-25	2	32
20-22	7	30	20-22	7	30
17-19	6	23	15 to19.5	6	23
14-16	0	17	**10.5 to15 (Median Class)**	**9 (fm)**	**17**
11-13	9	17			
8-10	7	8	8-10	7	**8 (cfb)**
5-7	1	1	5-7	1	1
	N = 34			N = 23	

Alternatively, we modify the class interval where N/2 may fall. The class interval with zero frequency which is affecting calculations is adjusted towards the adjoining class interval on either side and the size of the modified class interval is used while applying the formula. Half of the class interval 14-16 has been adjusted towards the class intervals 11-13 and 17-19 respectively. The modified class intervals are mentioned in terms of exact limits (modified size of class interval being 3 + 1.5 = 4.5).

$$Median = L + \frac{\frac{N}{2} - cfb}{fm} \times \text{c.i.}$$

L = 10.5

cfb = 8

N/2 = 17

fm = 9

$$Median = 10.5 + \frac{17 - 8}{9} \times 4.5$$

= 10.5 + 4.5 = 15 (Answer)

*(**Source:** The special cases for calculation Median has been taken from, ES-333, IGNOU, 2010)*

14.6.3 Use of Median

Median is used in the following situations :

- If few scores are not known.
- When the point dividing the distribution into two equal parts is needed.
- When a distribution is markedly skewed.

14.6.4 Limitations of Median

The major limitations of Median are as follows:

- Individual scores are not considered in calculating the Median. Even in the absence of an individual score, Median can be calculated.
- Median is not an accurate measure of central tendency.
- It can not be used as the centre of gravity of the distribution.
- It can not be used for inferential statistic analysis.

Check Your Progress 2

Note: a) Write your answers in the space given below.

b) Compare your answers with those given at the end of the Unit.

5. Define Median.

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6. Median is related to which scales of measurement?

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14.7 THE MODE

Mode as a measure of Central Tendency is used when the data are in nominal scale. The data obtained in nominal scale is of classificatory type and mostly qualitative in nature. We can classify the data, make categories in different heads and obtain frequencies. Mode is simply defined as the 'most frequently occurred item in the series'. For example, let 30 students are in a group, and in a test of Mathematics, the score and frequencies of the students are as follows:

Score	Frequencies
30	1
35	2
37	3
40	4
46	2
53	7
59	3
62	3
67	2
70	1
72	2
	N = 30

From the above distribution, among the 30 students, we find that seven students have secured the score 53. As per the definition of Mode, the score 53 which occurred most frequently in the distribution, is the mode. From the above table, you might have observed that, when we compute Mode of a distribution, we simply ignore the other scores. It implies that Mode is not an accurate or reliable measures of central tendency as because each and every score values are not considered or contributed for computing Mode. Again, there is also the possibility of more than one Mode in a distribution. If frequency of two different scores is highest and same in the group, there will be the situation of two Modes in that distribution. In such case, we can say that the distribution is bi-modal or in case three Modes are there in the group, we can say it is as tri-modal distribution. It is possible that a distribution may have more that one Mode but there exist only one Median and Mode in a distribution. Now, let us discuss, how to compute Mode.

14.7.1 Calculation of Mode for Ungrouped Data

Calculation of Mode in an ungrouped data is called as empirical or crude Mode. As discussed above, Mode is calculated in an ungrouped data by tallying the frequencies of the scores or items. In a simple ungrouped set of scores, the mode is the score which occurs most frequently. For example, if the scores in a group of 15 students are 30, 32, 41, 45, 40, 32, 40, 45, 49, 57, 59, 60, 59, 45 and 65. The Mode of the distribution will be 45, which occurs most frequently i.e. thrice in the group. Accordingly, Mode can be calculated in ungrouped data.

There is the possibility of more than one mode in a distribution. For example, if the individual scores in a distribution is 20, 23, 15, 33, 41, 54, 38 33, 23, 11, 10, 23, 51, 41, 50, 33, 45, 42, 15 and 38. In this distribution, the frequency of the scores 23 and 33 is 3 each and that is also highest. So in this distribution there are two modes i.e. 23 and 33. This is called as a bi-modal class. Accordingly tri-modal class is also possible if there are three modes in a distribution.

14.7.2 Calculation of Mode for Grouped Data

Calculation of Mode in group data is called as the true mode. It can be calculated by employing direct as well as indirect methods. In indirect method, mode is calculated by using the following formula:

Mode in Indirect Method:

Mode = (3 × Median) – (2 × Mean)

i.e., for calculating Mode, there is need to calculate Median and Mean first and than to calculate Mode. Let us take an example and calculate Mode in indirect method.

Example: Calculate Mode of the following distribution.

Class Interval	90-99	80-89	70-79	60-69	50-59	40-49	30-39	20-29	10-19
Frequencies	2	3	4	7	10	6	5	2	1

Solution:

Class Interval (C.I.)	**Mid-point**	**Frequencies (f)**	**Cumulative Frequency (cf)**	**Deviation (d)**	**f d**
90-99	94.5	2	40	+ 4	8
80-89	84.5	3	38	+ 3	9
70-79	74.5	4	35	+ 2	8
60-69	64.5	7	31	+ 1	7
50-59 (Median Class)	**54.5 (A.M.)**	**10**	**24**	**0**	**0**
40-49	44.5	6	14	- 1	- 6
30-39	34.5	5	8	- 2	- 10
20-29	24.5	2	3	- 3	- 6
10-19	14.5	1	1	- 4	- 4
		N = 40			$\sum fd = 6$

Median in Short Method:

$$Median = L + \frac{\frac{N}{2} - cfb}{fm} \times \text{c.i.}$$

L = 49.5

cfb = 14

N/2 = 20

fm = 10

c.i. = 10

$$Median = 49.5 + \frac{20 - 14}{10} \times 10$$

= 49.5 + 6 = 55.5

Mean in Assumed Mean Method:

$$Mean\ (M) = A.M. + \frac{\sum fd}{N} \times c.i.$$

A.M. = 54.5

N = 40

c.i. = 10

$\sum fd$ = 6

$$Mean\ (M) = 54.5 + \frac{6}{40} \times 10$$

= 54.5 + 1.5

= 56

Mode = (3 × Median) – (2 × Mean)

= (3 × 55.5) – (2 × 56)

= 166.5 – 112

= 54.5 (Answer)

Mode in Direct Method:

In a grouped frequency distribution, it is unrealistic to calculate the Mode by looking only the frequencies. A direct technique therefore needs to be employed to calculate the Mode. For this, the following method can be used to calculate Mode. f_1

$$Mode = L + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times c.i.$$

Where, L = Lower Limit of the Modal Class

c.i. = Size of the class interval

f_1 = Frequency of the modal class

f_0 = Frequency of the class preceding of the modal class

f_2 = Frequency of the class succeeding of the modal class

*(**Source**: NCERT, 2010)*

Let us calculate Mode in direct method by using the above example.

Example:

Calculate Mode of the following data in direct method.

Class Interval	90-99	80-89	70-79	60-69	50-59	40-49	30-39	20-29	10-19
Frequencies	2	3	4	7	10	6	5	2	1

Solution:

Class Interval (C.I.)	Frequencies (f)
90-99	2
80-89	3
70-79	4
60-69	**7 (f_2)**
50-59 (Modal Class)	**10 (f_1)**
40-49	**6 (f_0)**
30-39	5
20-29	2
10-19	1
	N = 40

Step 1 : Identification of the Modal Class [The class having the highest frequency, i.e. 50-59]

Step 2 : Lower Limit of the Modal Class 50-59 is 49.5

Step 3 : Calculation of 'f_1' (is the frequency of the Modal Class, i.e. 10)

Step 4 : Calculation of 'f_0' (is the frequency of just preceding the Modal Class, i.e. 6)

Step 5 : Calculation of 'f_2' (is the frequency of just succeeding the Modal Class, i.e. 7)

Step 6 : Size of the Class is 10.

$$Mode = L + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times c.i.$$

$$= 49.5 + \frac{10-6}{2\times10-6-7} \times 10$$

$$= 49.5 + \frac{4}{7} \times 10$$

$$= 49.5 + 5.71$$

$$= 55.21 \text{ (Answer)}$$

Now you can compare both the values of Mode, calculated by indirect method and direct method.

14.7.3 Use of Mode

Mode is used in the following situations:

- Mode is used, when it requires to find the most frequently occurred item as the measure of central tendency.
- When a quick and approximate measures of central tendency is required.
- When data is incomplete and skewed.

14.7.4 Limitations of Mode

The following are some of the limitations of Mode:

- Mode can only be used as a rough estimate of measures of central tendency, it can never be an accurate measure of central tendency.
- There may be possibility of more that one mode in a distribution, whereas, it is not possible for the case of Mean and Median.
- In case, all observations/scores in a group are different or there is no repetition of scores, it is difficult to get the Mode as all scores can be represented as Mode.
- Mode is only a crude measure which can be of value when a quick and rough estimate of central tendency is required.

Activity 2

Discuss with an example the possibility of more than one Modes in a distribution.

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Check Your Progress 3

Note: a) Write your answers in the space given below.

b) Compare your answers with those given at the end of the Unit.

7. Define Mode.

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8. Mode is related to which scales of measurement?

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14.8 COMPARISON OF MEAN, MEDIAN AND MODE

You have already discussed the concept, processes of calculation, uses and the limitations of Mean, Median and Mode in the previous sections. In this section, we will compare the three.

Table 14.2: Comparison of Mean, Median and Mode

Aspects	Mean	Median	Mode
Nature of Data	Equal Interval Scale	Ordinal Scale	Nominal Scale
Concept	Mean is a point on the scale of measurement obtained by dividing the sum of all the scores by their total numbers.	Median is the mid-point of the distribution, below and above which 50 percent of the cases lie.	Mode is the score of the distribution which occurs most frequently.
Accuracy	Mean is the most accurate measure of central tendency.	Median is a approximate measure of central tendency.	Mode is a rough estimate of measure of central tendency.
Use	Only when all the scores of the distribution are known and distribution is symmetrical.	Can be calculated in case few scores are not known and if the distribution is skewed.	Can be calculated in case the absence of few scores and if the distribution is skewed.
Values	There will be only one Mean in the distribution.	There will be only one Median in the distribution.	There may be more than one Mode in the distribution.

Activity 3

Critically analyse the use of Mean, Median and Mode with suitable examples.

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14.9 SELECTION OF AN APPROPRIATE MEASURE OF CENTRAL TENDENCY

Selection of an appropriate average requires experience and insight to examine the nature of data and the purpose in hand. A careful study of the various sections of this chapter would guide you to select appropriate statistics to calculate the measures of central tendency. The scale of measurement on which data are available also plays an important role in selecting an appropriate average. By converting data from one scale to an other, different measures of central tendency can be used. The purpose of using the average should be kept in mind while

selecting a measure of central tendency. For the research purpose, mean is popularly used as measure of central tendency as it the most accurate measure of central tendency.

14.10 LET US SUM UP

In this Unit, you studied about various methods of calculating measures of central tendency. In a lay person language, when we talk about average score it implies that to add all the scores secured by the students in a group and divide it by the total number of students. But when we elaborately study various methods of calculating the measures of central tendency, we find that for calculating Mean, Median and Mode, the formula are different even techniques are also different to calculate the measures of central tendency for ungrouped and grouped data. You have also studied, how the nature of the data determines the use of statistics and the scale of measurement that it deals with. It is therefore, one has to take a conscious decision about the use of various measures of central tendency and also their applicability.

14.11 REFERENCES AND SUGGESTED READINGS

IGNOU (2010). Measures of Central Tendency (Unit 13; Block 4), Educational Evaluation (ES-333, B.Ed.), New Delhi: IGNOU.

Garrett, H.E. (1973). Statistics in Psychology and Education, Vakils, Feffer and Simons Pvt. Ltd., Bombay.

Guilford, J.P. (1965). Fundamental Statistics in Psychology and Education, McGraw Hill Book Company, New York.

Nayak, B.K. and Rath, R.K. (2010). Measurement, Evaluation, Statistics and Guidance Services in Education, New Delhi: Axis Publications.

14.12 ANSWERS TO CHECK YOUR PROGRESS

1. (c) Equal interval scale
2. (a) Symmetrical
3. (b) Each score is given weightage to calculate Mean
4. (d) All the above
5. Median is the mid-point of the distribution, below and above which 50 percent of the cases lie.
6. Ordinal scale
7. Mode is defined as the most common item in the series.
8. Nominal scale