UNIT 15 MEASURES OF DISPERSION

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15.1 INTRODUCTION

In the previous units, you must have studied about tabulation of graphical representation of data and measures of central tendency. The measures of central tendency is after all a single numerical value and may fail to reveal the data entirely. Thus, the next step to measures of central tendency is to know about measures of dispersion. In this Unit, you will understand the concept of dispersion or variability among the data. Variability may be understood simply in the following lines.

"Mean as a measure of central tendency describes only one of the important characteristics of a distribution of scores but, it is equally important to know how compactly the scores are distributed from this point of location or conversely how far they are scattered away from it, the latter explains the concept of variability."

For example, the mean of the following two sets of scores are equal i.e. 10, however the spread of scores are different in two groups, even the range of highest and lowest scores is also different.

Group I : 8, 12, 11, 12, 10, 8, 9, 11, 12, 10, 8, 10, 9, 10, 12, 8, 10, 9, 10 and 11.

Group II : 15, 2, 8, 12, 4, 17, 20, 6, 2, 18, 16, 0, 3, 9, 6, 10, 15, 17, 9 and 11.

In the above example, the two groups are not comparable in terms of homogeneity, even if the mean scores of two groups are the same. The dispersion or the variability of the scores of the groups are different.

In the present Unit, we will discuss measures of dispersion and measures of position, its calculation, interpretation and use in classroom situation. Mastery on such concepts will make you able to understand the spread and variability of abilities of the students in your class individually as well as in a group.

15.2 OBJECTIVES

After going through the Unit, you will be able to:

- understand the measures of position;
- differentiate between percentile and percentile rank;
- calculate the percentile and percentile rank;
- interpret the obtained values of percentile and percentile rank;
- understand the concept of dispersion;
- state the importance of the measures of dispersion;
- define and calculate different measures of dispersion viz. – range, quartile deviation, mean deviation, and standard deviation; and
- use appropriate measures of dispersion according to the need of classroom situations.

15.3 MEASURES OF POSITION

The purpose of Statistics is to help you understand the data. Therefore, until we know the position of a particular score in any group, that score remain meaningless for us, e.g., if Rajesh got 85 marks in Physics out of 100, the score informs you of the achievement of Rajesh, but if you want to judge the ability of Rajesh in Physics, then you need to know his relative position in the group; may be after securing 85 marks he obtained last position in the group, all other members of the group might have secured more than 85 marks or else Rajesh might be standing in the middle or may be at the top position. Therefore, to know the ability of Rajesh with respect to his class, it is necessary to know that among all scores, what is the position of 85. For example, if percentage of students who scored less than 85 marks is 20%, then it is clear that Rajesh is one among the below 20% achievers. Those scores which possess specific position in the group are known as measures of position.

15.4 PERCENTILE AND PERCENTILE RANK

There are several measures of position e.g. Percentile, Decile, Quartile, etc., but in this Unit, you will study the commonly used measures of position i.e. percentile and percentile rank.

15.4.1 Concept of Percentile and Percentile Rank

In measures of central tendency you have studied about median. Median is the midpoint of the series, also we can say that median is that point in a frequency distribution below and above which 50% of the cases lie. Similarly first quartile i.e. Q_1 and third quartile i.e. Q_3 are those points below which lie 25% and 75% measures respectively. Similarly, you can calculate the points below which any percent of score e.g. 10%, 15%, 35%, 67%, 85%, 99%, etc. lies. These points are known as percentiles and we may represent the percentile using notations viz., P_{10}, P_{15}, P_{35}, P_{67}, P_{85}, P_{99}, etc. respectively.

Similarly, we may define percentile and percentile rank in the following words:

Percentile	The Kth percentile of a given score in any distribution is the point on the score scale below which 'K' percent of the scores fall.
Percentile Rank	The percentile rank of a given point on a score scale is the percentage of measures in the whole distribution which are below that given points.

15.4.2 Calculation of Percentile and Percentile Rank

i. Percentile

In order to calculate the values of percentile, one needs to locate the points on the scale of measurement upto which the given percent of cases lie. For calculating percentile, the formula which we use to calculate median are used.

The formula is:

$$P_p = l + \left(\frac{pN - Cf_B}{f_p} \right) \times i \quad \text{........ (1)}$$

Where,

P = Percentile

p = Percentage of distribution desired e.g. 20%, 35%, etc.

l = Exact lower limit of the class interval upon which Pp lies

pN = Part of N

Cf_B = Cumulative Frequency below l

Example: Calculate P_{10}, P_{20}, P_{30}, P_{40}, P_{50}, P_{60}, P_{70}, P_{80}, P_{90}, for the following data:

Table: 15.1

Scores	*f*	*c.f.*
10 – 19	2	2
20 – 29	4	6
30 – 39	5	11
40 – 49	10	21
50 – 59	35	56
60 – 69	20	76
70 – 79	13	89
80 – 89	8	97
90 – 99	3	100
	N = 100	

Solution:

P_{10} = 10% of 100 = 10 → = $29.5+\left(\frac{10-6}{5}\right)\times 10$ = 37.5

P_{20} = 20% of 100 = 20 → = $39.5+\left(\frac{20-11}{10}\right)\times 10$ = 48.5

P_{30} = 30% of 100 = 30 → = $49.5+\left(\frac{30-21}{35}\right)\times 10$ = 52.07

P_{40} = 40% of 100 = 40 → = $49.5+\left(\frac{40-21}{35}\right)\times 10$ = 54.92

P_{50} = 50% of 100 = 50 → = $49.5+\left(\frac{50-21}{35}\right)\times 10$ = 57.78

P_{60} = 60% of 100 = 60 → = $59.5+\left(\frac{60-56}{20}\right)\times 10$ = 61.5

P_{70} = 70% of 100 = 70 → = $59.5+\left(\frac{70-66}{20}\right)\times 10$ = 66.5

P_{80} = 80% of 100 = 80 → = $69.5+\left(\frac{80-76}{13}\right)\times 10$ = 72.57

P_{90} = 90% of 100 = 90 → = $79.5+\left(\frac{90-89}{8}\right)\times 10$ = 80.75

Thus, the value of P_{10}, P_{20}, P_{30}, P_{40}, P_{50}, P_{60}, P_{70}, P_{80}, P_{90}, is 37.5, 48.5, 52.07, 54.92, 57.78, 61.5, 66.5, 72.57, 80.75 respectively.

ii. Percentile Rank (PR)

You have seen, how percentiles e.g., P_{10} to P_{90} have been calculated directly from any given frequency distribution and based on the above calculation we may say that, "percentiles are points in a continuous distribution below which lie given percentages of N". Now you can calculate the problem of finding an individual's percentile rank (PR). The following formula may be used for calculating the PR :

$$PR = \frac{l00}{N} + \left(Cf_B + \frac{X-l}{i} f\right) \quad \text{........ (2)}$$

Where,

X = Score for which PR is to be calculated.

Cf_B = Cumulative Frequency below l

l = lower limit having X

f = frequency of C.I. having X

i = Class Interval

N = Total frequency

Example

Calculate the PR for the score 35 and 55 for the distribution of scores given in Table '15.1'.

(i) for X = 35;

X = 35, Cf_B = 6, l = 29.5, f = 5, i = 10, N = 100

$$PR = \frac{100}{100} + \left(6 + \frac{(35 - 29.5)}{10} \times 5\right)$$

= 8.75

∴ PR of 35 is 8.75

(ii) for X = 55;

X = 55, Cf_B = 21, l = 49.5, f = 35, i = 10, N = 100

$$PR = \frac{100}{100} + \left(21 + \frac{(55 - 49.5)}{10} \times 35\right)$$

= 40.25

∴ PR of 55 is 40.25

15.4.3 Interpretation of Percentile

Percentile is a set of measure used to indicate the relative position of a single item of individual in context with the group to which the item of individual belongs. In other words, it may be said that it is used to indicate the relative position of a given score among other scores. As you have already studied that percentile refers to a point in a distribution of scores or values below which a given percentage of cases lie, therefore 85% of the observations are below the 85th Percentile, which may be denoted as P_{85}.

In testing and interpreting the test scores, percentiles are very useful. Whenever you want to compare two individuals, it is always better and advised to compare them on the basis of Percentile Rank and not on the basis of scores they have secured. For any standardized tests, percentile norms are given with the test and, therefore, with the help of norms one may interpret the results properly.

15.4.4 Limitations

In spite of having certain important and relevant characteristic of knowing the ability of an individual in any group based on their achievement scores, the percentile has some limitations which you need to understand, viz., the mastery of an individual may not be judged by the percentile, because the same person in a poor group may perform better and may secure good rank, while the person in a better group may perform poor and may secure poor rank.

Check Your Progress 1

Note: a) Write your answer in the space given below.

b) Compare your answer with those given at the end of the Unit.

1) (i) Define Percentile and Percentile Rank.

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(ii) Calculate Percentile Rank of 34 for the following scores given below:

Scores Interval	70–79	60–69	50–59	40–49	30–39	20–29	10–19	0–9
f	4	3	5	6	3	5	2	2

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15.5 MEASURES OF DISPERSION

You have already studied that the purpose of statistics is to describe the characteristics of any variable with reference to any group. Measures of central tendency provide some meaningful information on any data but it is not at all sufficient to make a holistic and comprehensive view e.g. if there are three groups having eight students in each who secured the following marks on any achievement test:

A:	20	20	20	20	20	20	20	20
B:	10	14	18	20	10	52	20	16
C:	20	20	20	40	60	00	00	00

It is evident from the data that 'mean score' for all the three groups are the same, but whether the groups are also the same? Think! Equal mean score i.e. 20 for all the three data sets informs you that all the members of a group are varying around '20', but when you see the group scores individually, you find a significant variation in scores, and this variation is because of spreadness or so to say dispersion. Therefore, in order to make any concrete decision for the group, you need to know the dispersion that exists among the data. This dispersion may be known by applying the any of the following measures:

i. Range

ii. Quartile Deviation

iii. Mean Deviation

iv. Standard Deviation

15.5.1 Range

Range may be defined simply as that interval between the lowest and the highest scores. It is very common and general measure of spread and is computed when we need to know at a glance comparison of two or more groups for variability. Since, it is based on two extreme values and tells nothing about the variation of the intermediate values, it is not an authentic measure of dispersion. Range may be computed by the following formula:

(Range = Highest Score – Lowest Score) .... (3)

It is deceptive and not authentic. For example, in any class of 40 students, 1 student got 20 marks, while all the rest 39 students scored between 70 to 80 marks out of 100. Range informs you that there exist a range of 60 marks, while the majority secured between 70 to 80 and the more appropriate and near variation is of 100 marks only. Therefore, it may be used as a quick and at a glance measure of dispersion, but you cannot be dependent on "Range" in order to know true dispersion.

15.5.2 Quartile Deviation and its Interpretation

The second measure of measures of dispersion is Quartile Deviation. It is also known as 'semi-interquartile range'. As you know the interval between highest and lowest score is known as 'range', in a similar way distance between first and third quartile divided by two is known as 'Quartile Deviation'. Therefore, it may be expressed as:

$$\text{QD} \quad \text{or} \quad Q = \frac{Q_3 - Q_1}{2} \qquad \text{........ (4)}$$

Since you are already aware on the concept of percentile, therefore, you may simply infer the formula in the following manner:

$$\text{QD} \quad \text{or} \quad Q = \frac{P_{75} - P_{25}}{2} \qquad \text{........ (5)}$$

In order to calculate Q, it is clear that we must first compute the 75^{th} and 25^{th} Percentile and, therefore, the formula (1) previous discussed for calculating percentile may be used in the following manner:

$$Q_1 = l + \left(\frac{\frac{N}{4} - Cf_B}{f_q} \right) \times i \qquad \text{........ (6)}$$

and

$$Q_3 = l + \left(\frac{\frac{3N}{4} - Cf_B}{f_q} \right) \times i \qquad \text{........ (7)}$$

Note: You must remember that formula (1), used for Percentile has been taken from the formula which you used in calculating the median.

Example: Calculate the Quartile Deviation for the data given in Table 15.1.

Solution: Using the data given in Table 15.1 and Formula (6) and (7):

(i) $Q_1 = P_{25} = 49.5 + \left(\frac{25 - 21}{35} \right) \times 10 = 50.64$

(ii) $Q_3 = P_{75} = 59.5 + \left(\frac{75 - 56}{20} \right) \times 10 = 69$

Therefore, $\text{QD} = \frac{Q_3 - Q_1}{2} = \frac{69 - 50.64}{2} = 9.18$

Q = 9.18

Interpretation of Quartile Deviation

Quartile deviation is easy to calculate and interpret, it is independent of the problem of extreme values and, therefore, it is more representative and authentic than range. In distribution where we prefer median as a measure of central tendency, the quartile deviation is also preferred as measure of dispersion. However, both the measures are not suitable to algebraic operations, because both do not consider all the values of the given distribution. In case of symmetrical distribution, mean and median are equal and median lies at an equal distance from the two quartiles i.e.

Q_3 – Median = Median – Q_1

In case of non-symmetrical distribution, two possibilities may arise:

I. Q_3 – Median > Median – Q_1 (Positive Skewed Curve)

II. Q_3 – Median < Median – Q_1 (Negative Skewed Curve)

15.5.3 Mean Deviation and its Interpretation

The mean deviation (MD), also known as average deviation (AD), is the mean of all deviations of all individual and separate scores in a distribution taken from the mean. While calculating mean deviation, no account is taken of signs and, therefore, all deviations whether having plus or minus sign are treated as positive only.

The following formulae may be used to calculate the Mean Deviation (MD):

$$MD = \frac{\sum |x|}{N}$$ (For ungrouped data) .... (8)

$$MD = \frac{\sum |fx|}{N}$$ (For grouped data) .... (9)

Where, (x = X – M)

Example: Calculate the mean deviation for the distribution given in Table 15.1 of this unit:

Scores	***X (Mid-point)***	***f***	***fX***	$\lvert x\rvert = \lvert X - M\rvert$ (M=58.7)*	$\lvert f x\rvert$
10 – 19	14.5	2	29	44.2	88.4
20 – 29	24.5	4	98	34.2	136.8
30 – 39	34.5	5	172.5	24.2	121
40 – 49	44.5	10	445	14.2	142
50 – 59	54.5	35	1907.5	4.2	147
60 – 69	64.5	20	1290	5.8	116
70 – 79	74.5	13	968.5	15.8	205.4
80 – 89	84.5	8	676	25.8	206.4
90 – 99	94.5	3	283.5	35.8	107.4
		N=100	$\sum fX = 5870$		$\sum \lvert f x\rvert$ = 1270.40

* Calculation of Mean Score you have already learned in Unit-14 of this Block.

$$\text{MD} = \frac{\sum |fx|}{N} = \frac{1270.40}{100} = 12.704$$

Therefore, **MD = 12.704**

Interpretation of Mean Deviation

Unlike ‘Range’ and ‘Quartile Deviation’, the mean deviation is the simplest measure of dispersion that takes into account all the scores of distribution. In spite of having this feature, mean deviation is rarely used in modern statistics, however, you will find it in the older literature. Because this method, it ignores the importance of plus and minus sign. Therefore, the method doesn’t fulfill the assumptions of algebraic properties and hence it is not possible to use this method in higher statistics.

Check Your Progress 2

Note: a) Write your answer in the space given below.

b) Compare your answer with those given at the end of the Unit.

2. (i) Define Average Deviation.

(iii) Calculate Quartile Deviation for the data given below:

C.I	190-199	180-189	170-179	160–169	150–159	140–149	130–139	120–129	110–119
f	6	14	22	36	42	15	8	6	4

15.5.4 Standard Deviation

Standard deviation (SD) is commonly and frequently used measure of dispersion, because SD is the most consistent and stable index of variability and is usually used in experimental research. Deviations of all scores taken from their mean and then square root of their average, is known as, Standard Deviation. The standard deviation differs from mean deviation in many respects. In calculating the mean deviation, we ignore signs and treat all deviations as positive, whereas in standard deviation we avoid such complexities of signs by squaring the individual deviations. Also, unlike mean deviation, in the standard deviation, the squared deviations used in the process are always taken from the mean only, never from the median or mode. The commonly acknowledged symbol for the standard deviation is the Greek letter ‘σ’ (known as Sigma).

In conclusion, it may be said that ‘The sum of the squared deviations from the mean, divided by N, is known as the variance and the square root of the variance is known as standard deviation’.

Following are the formula, which you may use according to your convenience and the nature of the data:

a) $$\sigma = \sqrt{\frac{\sum x^2}{N}} \qquad \text{........ (10)}$$

where (x = X – M) (For ungrouped data)

b) $$\sigma = \sqrt{\frac{\sum fx^2}{N}}$$ .... (11)

where (x = X – M) (For grouped data)

(**Note:** This formula is generally used when value of mean is in whole number).

c) $$\sigma = \sqrt{\frac{\sum fx^2}{N} - \left(\frac{\sum fX}{N}\right)^2}$$ .... (12)

(**Note:** This formula is generally used when value of mean is in fraction).

d) $$\sigma = i\sqrt{\frac{\sum fd^2}{N} - \left(\frac{\sum fd}{N}\right)^2}$$ .... (13)

(**Note:** This formula is generally used as short method for calculating S.D.).

Where,
- f : Frequency
- X: Raw Data
- x: deviated value from mean
- d: deviation from assumed mean
- i: class-interval
- N: Total frequency

Now, we will learn the computation of standard deviation using formula 10, 11, 12 and 13.

Example: (For ungrouped data)

(1) Calculate the Standard Dispersion for the following data:

7, 8, 11, 14, 15

Solution:

X	***x = X – M***	***x²***
7	7 – 11 = – 4	16
8	8 – 11 = – 3	9
11	11 – 11 = 0	0
14	14 – 11 = 3	9
15	15 – 11 = 4	16
$\sum X = 55$	$\sum X = 0$	$\sum x^2 = 50$

$$\sigma = \sqrt{\frac{\sum x^2}{N}}$$

$$= \sqrt{\frac{50}{5}} = \sqrt{10}$$

$$\sigma = 3.16$$

Example : (For grouped data)

2. Calculate the standard deviation for the distribution given in Table 15.1 of this Unit.

Example: Calculate the mean deviation for the distribution given in Table 15.1 of this unit:

Scores	***f***	***X* (Mid-point)**	***fX***	$f.X^2$	$x = X - M$	$x^2 = (X-M)^2$	$fx^2 = (X-M)^2$
10 – 19	2	14.5	29	420.5	44.2	1953.64	3907.28
20 – 29	4	24.5	98	2401	34.2	1169.64	4678.56
30 – 39	5	34.5	172.5	5951.25	24.2	585.64	2928.2
40 – 49	10	44.5	445	19802.5	14.2	201.64	2016.4
50 – 59	35	54.5	1907.5	103958.75	4.2	17.64	617.4
60 – 69	20	64.5	1290	83205	5.8	33.64	672.8
70 – 79	13	74.5	968.5	72153.25	15.8	249.64	3245.32
80 – 89	8	84.5	676	57122	25.8	665.64	5325.12
90 – 99	3	94.5	283.5	23790.75	35.8	1281.64	3844.92
	N = 100		$\sum fX =$ 5870	$\sum fX^2 =$ 371805			$\sum fx^2 =$ 27236

Calculation of Standard Deviation using different formulas.

A. Formula 11.

$$\sigma = \sqrt{\frac{\sum fx^2}{N}}$$

$$= \sqrt{\frac{27236}{100}}$$

$$\sigma = 16.50$$

B. Formula 12.

$$\sigma = \sqrt{\frac{\sum fx^2}{N} - \left(\frac{\sum fx}{N}\right)^2}$$

$$= \sqrt{\frac{371805}{100} - \left(\frac{5870}{100}\right)^2}$$

$$= \sqrt{3718.05 - 3445.69}$$

$$\sigma = 16.50$$

Example: (Using Short Method)

Calculate the Standard Deviation for ‘Table 15.1’ given in this Unit, using short method.

15.5.5 Sheppard's Correction

When σ is computed from a frequency distribution, the scores in each interval are represented by the midpoint of that interval. Fair possibility of equal distribution and symmetry cannot be ensured every time and, therefore, frequencies tend to lie below the midpoint more often than above in the intervals above the mean of the distribution whereas in intervals below mean, the scores tend to lie above the midpoints. These errors may be taken care of if mean is calculated from all the intervals. But the 'grouping error' may inflate σ and situation may become worst, if the intervals are wide and N is small. In order to prevent the error and adjust for grouping 'Sheppard's Correction' is frequently used. The formula for Sheppard's correction is as follows (the value 12 used in this formulla is a constant):

$$\sigma = \sqrt{\sigma^2 - \frac{i^2}{12}} \qquad \text{.... (14)}$$

For example, if you apply this formula for the aforesaid result, it will be:

$$\sigma = \sqrt{(16.50)^2 - \frac{(10)^2}{12}}$$

$$\sigma = \sqrt{272.25 - 8.53}$$

$$\sigma = 16.24$$

∴ Value of SD after applying correction formula is 16.24.

15.5.6 Interpretation of Standard Deviation

Standard deviation is commonly and widely used statistics in data analysis and research. Like mean deviation, it utilizes all the values of the distribution. This is called as accurate measure of dispersion because as it gives equal weightage to each and every score in the series. Standard deviation may be seen as spine of statistics, it is the master measure of dispersion and amenable to algebraic operations and also we use it in corelational studies and in further statistical analyses like ANOVA and ANCOVA. The standard deviation is less affected by sampling errors than Q and MD.

Check Your Progress 3

Note: a) Write your answers in the space given below.

b) Compare your answers with those given at the end of the Unit.

3) (i) Define the relation between 'Variance' and Standard Deviation'.

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(ii) Calculate Standard deviation for the following set of scores:

40, 38, 42, 60, 72, 54

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15.6 USE OF MEASURES OF POSITION AND DISPERSION

Needless to mention the important and significant role of 'assessment' in progress and advancement of student as an individual and classroom and institutions as a group, also needless to highlight the role of statistics in the process of assessment of a student and a class. In classroom situation the need of assessment has been highlighted by almost all the committees and commissions set up in independent India. It is a commonly acknowledged fact that we need to use some basic statistics in classroom situation frequently to know the relative and comparative progress of a student as well as a class over a period of time. However, majority of the time, it has been observed that the teacher uses the simplest measures of central tendency to highlight students' achievement and tries to avoid further analysis which can give an overall and clear-cut picture of the progress of an individual student in the class and of the class as a whole.

The following points may be kept in mind:

- In a classroom situation a teacher must calculate the achievement of a student using suitable measures of central tendency but the teacher must put the individual scores in the form of percentiles and percentiles ranks to know the achievement along with the student's relative position in the class.
- There are various measures of dispersions, Range, Quartile deviation, mean deviation and standard deviation and often the question arises which one is to be used in the classroom situation. Needless to mention the strength of standard deviation, however, other measures are also to be used, keeping in view the need of assessment. A brief description is given here to understand the use of various measures of dispersion.

Range:

(i) When you need quick, crude and at a glance measure.

(ii) When the scores spread widely.

(iii) When you need information from extremes only.

Quartile Deviation:

(i) When median has been used as a measure of central tendency.

(ii) When score is lying both sides of central score are of prime importance.

(iii) When extreme scores are affecting the Standard Deviation unexpectedly.

Mean Deviation

In general, it is avoided because mean deviation ignores the importance of sign and, therefore, it lacks algebraic properties.

Standard Deviation

Strength and effect of SD is well known to us, because it satisfies the assumptions of algebraic operations SD may be used:

(i) When you need to know the most reliable measure of variability.

(ii) When mean has been used as measure of Central tendency.

(iii) When you want to consider all the scores according to their size.

(iv) When you want to use higher statistics, viz. Correlation, ANOVA, ANCOVA, etc.

These measures of dispersion informs a teacher of the variability in class and hence it helps the teacher to prepare personalized instruction to address the need of deprived students. In order to be more authentic and comprehensive three important concepts viz. standard scores, the coefficient of variation and variance are being discussed here as follows:

Standard Scores

The raw scores of any text are no more than simple arithmetic scores having no meaning in itself until and unless it is compared in some frame of reference. This frame of reference we get after calculating mean and SD of the whole group. Therefore, in order to give meaning to raw scores, it needs to be changed in some standard score. Standard scores carry their meaning in themselves and comparisons between different individuals are possible with them. The commonly known standard scores are : Z=score and T-Score.

Z-Score

Z-Score is the oldest and commonly known standard scores. Z-Scores are the scores taken from the deviation of each score from mean and divided by SD. Therefore, it may be expressed as:

$$Z = \frac{X - M}{\sigma}$$

Where, X – Raw Score, M – Mean, σ – SD.

However, Z-Score provides sufficient information but keeping in view, few limitations of getting Z-Score positive or negative and its expression upto two point of decimal some people try to avoid it.

T-Score

In order to overcome the difficulties faced in Z-Score experts suggest T-Score. T-Score is basically a linear transformation of Z-Score. The word 'T' here used to pay respect to 'Thorndike'. You may calculate T-Score as follow:

$$T = 50 + 10\,Z$$

Coefficient of Variation

It is very useful in comparing the standard deviation of many groups exposed to similar test i.e. when many groups are given similar test and their means are almost equal, than with the help of CV the deviation between groups may be calculated. CV may be calculated as:

$$CV = \frac{\sigma}{M} \times 100$$

Variance

The square of SD is known as variance.

$$V = \sigma^2 = \frac{\sum (X - M)^2}{N} = \frac{\sum x^2}{N}$$

Keeping in view its suitability and fitness on algebraic properties it is mostly used in higher statistics.

15.7 LET US SUM UP

- Percentile is nothing but a sort of measure used to indicate the relative position of an individual with reference to the group to which he/she belongs.

- Commonly used measures of dispersion are Range, Quartile Deviation, Mean Deviation and Standard Deviation.
- Range is used when scores are spread widely and need quick, crude and at a glance measure.
- Quartile deviation is used when median has been used as a measure of central tendency.
- Standard Deviation is the most consistent and stable index of variability. SD is used when mean has been used as measure of central tendency.
- Raw scores are always compared with some frame of reference or standard scores. The commonly known standard scores are Z-score and T-score.

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15.9 ANSWERS TO CHECK YOUR PROGRESS

1. (i) A percentile is a point on the score scale below which a given percent of the cases lie. Whereas Percentile Rank is defined as the number representing the percentage of the total number of cases lying below the given score.

   (ii) N = 30, X = 34, L = 29.5, cf_b = 9, i = 10, f =3

   So, PR = 34.5

2. (i) Average Deviation is measured by the mean deviation of all separate scores in the series taken from their mean.

   (ii) $Q_1 = P_{25} = 150.62$

   $Q_3 = P_{75} = 171.54$

   So, Q.D. = $\frac{Q_3 - Q_1}{2} = 10.46$

3. (i) The square root of variance is known as Standard Deviation.

   (ii) $\Sigma x = 306$, N = 6, M = 51, $\Sigma x^2 = 902$

   Hence, SD =12.26