ISSN 0970-9282

The Primary Teacher

Volume XLI Number 1 January 2016

About the Journal

The Primary Teacher is a quarterly journal brought out by the National Council of Educational Research and Training (NCERT), New Delhi. The journal carries articles and researches on educational policies and practices, and values material that is useful for practitioners in contemporary times. The journal also provides a forum to teachers to share their experiences and concerns about the schooling processes, curriculum textbooks, teaching-learning and assessment practices. The papers for publication are selected on the basis of comments from two referees. The views expressed by individual authors are their own and do not necessarily reflect the policies of the NCERT, or the views of the editor.

OFFICES OF THE PUBLICATION DIVISION, NCERT

NCERT Campus
Sri Aurobindo Marg
New Delhi 110016 **Phone: 011-26562708**

108, 100 Feet Road
Hosdakere Halli Extension
Banashankari III Stage
Bengaluru 560085 **Phone: 080-26725740**

Navjivan Trust Building
P.O. Navjivan
Ahmedabad 380014 **Phone: 079-27541446**

CWC Campus
Opp. Dhankal Bus Stop
Panihati
Kolkata 700114 **Phone: 033-25530454**

CWC Complex
Maligaon
Guwahati 781021 **Phone: 0361-2674869**

Single Copy: ₹ ***65.00*** *Annual Subscription:* ₹ ***260.00***

Published by the Head, Publication Division, National Council of Educational Research and Training, Sri Aurobindo Marg, New Delhi 110 016 and printed at Chandra Prabhu Offset Printing Works (P) Ltd., C-40, Sector-8, Noida 201 301 (UP).

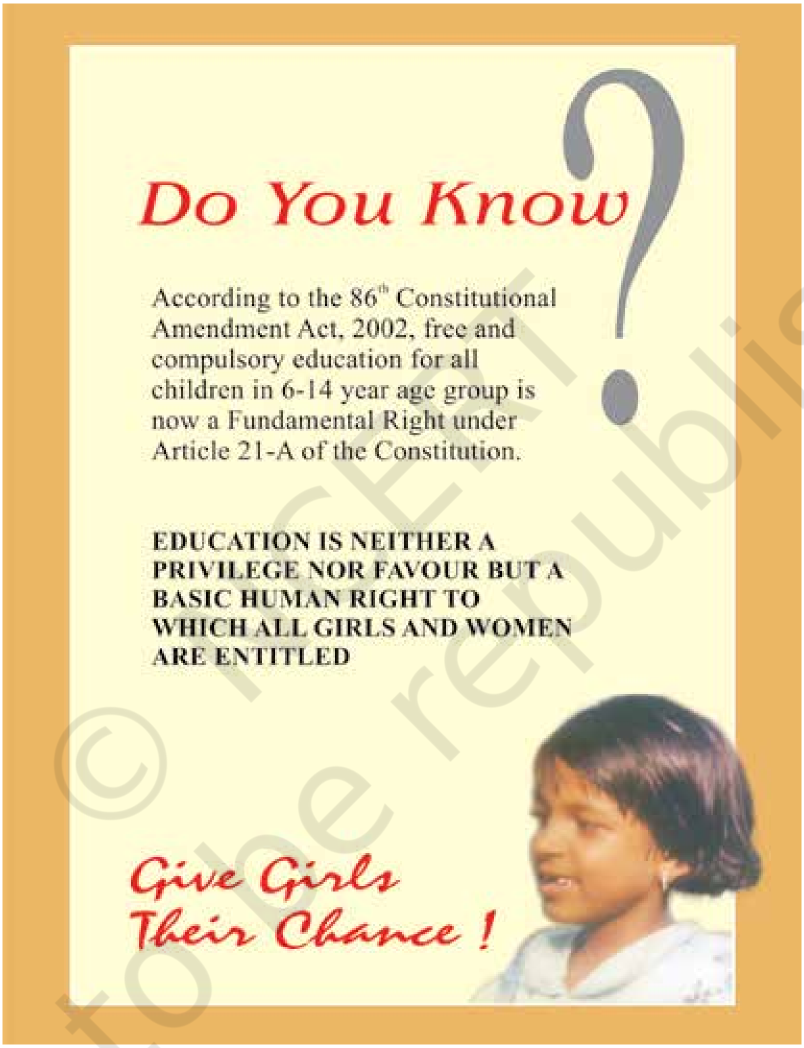
ISSN 0970-9282

The Primary Teacher

Volume XLI Number 1

January, 2016

CONTENTS


Do You Know?
According to the 86th Constitutional Amendment Act, 2002, free and compulsory education for all children in 6-14 year age group is now a Fundamental Right under Article 21-A of the Constitution.
EDUCATION IS NEITHER A PRIVILEGE NOR FAVOUR BUT A BASIC HUMAN RIGHT TO WHICH ALL GIRLS AND WOMEN ARE ENTITLED
Give Girls Their Chance !

EDITORIAL

The current issue of *The Primary Teacher* journal is dedicated to renowned Indian mathematician Dr. Srinivasa Ramanujan (22 December 1887–26 April 1920). His innate genius led to his independently compiling nearly 3,900 results of identities and equations most of which have been proven correct. His original and highly unconventional results, such as the Ramanujan prime and the Ramanujan theta function, have inspired numerous research works in the field of mathematics. In India, Ramanujan's birth anniversary is celebrated as the National Mathematics Day every year since 2011.

The first article focusses on questioning as a powerful instructional strategy. Teachers use questioning to check the understanding and knowledge of students to aid teaching, diagnose their difficulties, recall facts, direct attention and maintain control.

The second article dispels the myth that mathematics is a difficult subject and stresses that everyone can enjoy, engage and relax with mathematics.

'Learning from Errors in Numbers and Number Operations in Early School Mathematics' elaborates the need to re-conceptualise errors in the learning of the subject from obstacles to insights, to learners' thinking process and opportunities for learning. The paper discusses what error analysis means and how it can play an important role in integrating assessment with learning, as well as, help shift the focus from 'right' or 'wrong' answers to the wider meaning of learning in mathematics.

The paper titled 'Understanding Numbers — Concepts and Some Misconceptions' describes the importance of numbers in mathematics, as well as, in real life. In this paper, the authors discuss, with the help of examples, some problems faced by children at the primary level. There are certain points about numbers, which must be clear to teachers of the primary level.

The next paper discusses the outcome of an empirical study conducted on fourth and fifth grade students to find out the learning difficulties and the learning patterns they face or follow while working on word problems.

'Peer-learning in Mathematics among Primary School Children' describes the role of peer-learning on influencing a child's learning both cognitively and socially.

The experiences of students in their schools while engaging with rational numbers have been dealt with in another article, where the classroom experiences of the students are studied in tandem with the experiences explicated in the textbook.

The next article focusses on a study conducted to find out the anxiety faced by students of upper primary classes of Prakasam district in Andhra Pradesh while learning mathematics. The investigators have adopted normative survey method with a sample 200 students.

A laudable effort to humanise mathematics and mathematicians is the film, *The Man who knew Infinity*, a British biographical released in 2015. One of the articles in this issue deals with the attitude towards mathematics that it can be learnt by anyone, and that it is accessible even to a person from a 'non-math' background.

An article on mathematics laboratory emphasises on the largely deductive and abstract nature of the subject, which makes it appear dull and difficult. The structure of modern mathematical theories rest on those basic and elementary concepts, which come out of experiences with concrete objects.

A review of the book, *Alex's Adventures in Numberland,* brings out the fascinating side of the math world. Through intriguing and interesting anecdotes, it covers topics at the school level (Class V–XII), including arithmetic, algebra, geometry and statistics, presenting them in 12 chapters.

Interesting facts about the International Mathematical Olympiad (IMO) are also given, which are useful for students and teachers.

'My Page' examines exploration as a key activity in classroom, which can generate interest and learning about particular concepts and themes.

Anup Kumar Rajput
Varada M. Nikalje
Academic Editors

1

Questioning in teaching of Early Mathematics: A Review

Satya Bhushan*

"The teaching of mathematics should enhance a child's resources to think and reason, visualise and handle abstractions, formulate and solve problems."

— NCF, 2005

Abstract

Questioning is a powerful instructional strategy. Those interested in mathematics education have recognised the value of asking relevant questions for centuries. Classroom questioning is an extensively researched topic. Teachers use questioning fundamentally to check the understanding and knowledge of students to aid teaching, diagnose students' difficulties, recall facts, test their knowledge after the lesson is over, direct attention and maintain control. The high incidence of questioning as a teaching strategy and its consequent potential for influencing learning have led investigators to examine relationships between questioning methods and student achievement. The present review focuses on the importance of classroom questioning in the teaching-learning process in early mathematics.

Introduction

There is a growing body of evidence to indicate that early mathematics plays a significant role in education. From an analysis of six longitudinal studies, Duncan and colleagues have found that early mathematics skills are more powerful predictors of later academic achievement in mathematics and reading than attentional, socio-emotional or reading skills (Duncan, 2007, p. 1428). In addition, the differences in mathematical experiences that children receive in their early years "have long-lasting implications for later school achievement, becoming more pronounced during elementary

* *Assistant Professor*, Educational Survey Division, NCERT

school… and continuing on into middle school and high school" (Klibanoff, 2006, p. 59).

There are various learning theories in mathematics education. However, this paper focuses on constructivist theory for two reasons — one, school education in India is based on the National Curriculum Framework (NCF) – 2005, which endorses the constructivist approach; and two, questioning is one of the strategies followed in it. The constructivist theory expects students to be active in teaching-learning and teachers to guide the process. In constructivist teaching, a teacher's role is not to simply convey information, but to actively engage students in the process of acquiring knowledge. Teachers, who practise constructivist teaching, utilise various strategies to get students' views and understand their thinking. Questioning is one such strategy. When questions are used strategically by the teacher, socio-mathematical norms are established in the classroom. Teachers are able to evaluate students' thoughts. With this information, they can provide students with an opportunity to grapple with cognitively challenging problems as they guide them through the process of assimilation and accommodation in order to understand the problem. Questioning and discourse promote reasoning and intellectual development through social interaction. Teacher questioning assesses students' mathematical reasoning and provides needed information for scaffolding towards new understandings. In addition, questioning, requiring students' consistent engagement with constructivist theories, promotes student-centred learning.

Questions asked by a teacher that are related to ideas embedded in the curriculum will excite students' curiosity, promote critical thinking, elicit reflection and help them construct their own meanings for the mathematics they are studying. The responses will help the teacher assess what the students know and what the next instructional steps could be. Developing skills in questioning for understanding and content knowledge evolves over time and like anything else requires practice. The pay-off is significant in terms of students' conceptual understanding.

Research on the importance of questioning as a teaching and learning strategy is well documented (Almeida, Pedrosa de Jesus and Watts, 2008, Chin and Osborne, 2008; Graesser and Olde, 2003). It suggests that teachers spend up to 50 per cent of the class time on questioning and ask 300–400 questions a day (Levin and Long, 1981), while each student asks, on an average, one question per week (Graesser and Person, 1994). Surprisingly, teachers do not seem to be aware of this discrepancy. Several studies also rely on the kind of questions asked by teachers and students, concluding that these are, usually, procedural and fact-based (Brown and Edmondson,1985).

Characteristics of Questioning

Viewing one-to-one teaching of mathematics as an interactive communication is central to the literature being discussed here. In this section, four characteristics identified in the larger study are outlined — pre-formulating and reformulating questions, vague or ambiguous questioning, post-question wait-time, and questioning and prompting. A necessary feature of these characteristics is that they can be generalised across settings and tasks.

Pre-formulating and reformulating questions: Cazden (1986) cites the work of French and McClure (1981) to identify two interactive strategies, which serve as guidelines for children as they attempt to arrive at the answers teachers want. The first strategy is called 'pre-formulating'. Cazden reports that teachers, when pre-formulating questions, "preface a question they want the children to answer with one or more utterances, which serve to orient the children with the relevant area of experience and establish as shared knowledge between herself and the child and the materials essential to answer her question" (Cazden,1986). The second strategy is called 'reformulating'. Cazden argues that reformulating occurs when the initial answer is wrong. Reformulations vary depending on how the teacher makes the original question more specific. The important issue with reformulating is to what extent the teacher inadvertently or knowingly decreases the cognitive level of the task.

Vague or ambiguous questioning: According to Brophy and Good (1986), "Students sometimes cannot respond to questions asked by the teacher because the questions are vague or ambiguous, or because the teacher asks two or more questions without stopping to get an answer to the first one". A teacher's questions do not always get a response if they lack clarity or because s/he asks two or three questions without waiting for students' responses.

Wait-time: Wait-time is essential for student thinking. By wait-time, we refer to the time a teacher allots for student reflection after asking a question and before a student responds (wait-time I) and to the pause after the student has a responded (wait-time II). In her investigations, Rowe (1986) found that the mean wait-time was, on an average, one second or less. If a student did not answer in one second, the teacher would repeat or rephrase the question, ask another question or ask another student to respond. After receiving a response, the teacher waited for approximately 0.9 seconds before asking another question. Rowe trained the teachers to increase their wait-time to 3–5 seconds and found that both the quantity and quality of students' answers improved significantly. Students gave longer responses, cited more evidences to support their ideas, drew conclusions, speculated and hypothesised more. Besides, more students participated in the process. The students asked more questions and talked more with each other.

Questioning and prompting: According to Lyons, Pinnell and Deford (1993), questioning and prompting take much practice and experience. They found that a teacher of mathematics recovery, an early intervention programme for students who are 6–7 years old and in their second year of schooling (Wright, 1994), becomes more aware of a child's learning and previous experience and micro-adjusts her/his teaching accordingly. In any case, a teacher needs to be sensitive to a child's learning and make crucial decisions based on her/his observations of students.

Types of Questions

According to the NCERT (2010), various studies have categorised the types of questions that teachers ask in the classroom. Some of these categories are summarised below. Each item lists a question type, giving a brief description and examples.

Gathering information: These questions are, usually, closed and may involve checking students' factual or procedural knowledge, checking for a method, leading students through a method, or rehearsing by asking students to state known facts or procedures. For example, by using different shapes, a picture is drawn on a blackboard. The teacher asks questions like—How many triangles are there in the picture? What is more—triangles or squares?

Introducing or recalling terminology: These questions are useful when ideas are under discussion and the teacher wants the students to use correct mathematical language to talk about them. For example, in Class II, the children try to explain why when you add 43 + 4, you cannot add 4 + 4 and get 8. The children say that a single digit number should always be written on the right. The teacher asks, "What does '4' in '43' mean?" She wants them to recall the place value terminology and realise that adding 4 tens and 4 ones does not give 8 tens.

Probing: These questions are aimed at getting students to explain, clarify or elaborate their thinking for their own benefit and for the class. For example: When asked what is 6 + 4, a child says 10. Probing questions could be like: How did you get 10? Can you explain your idea?

Exploring mathematical meanings and relationships: These questions point to underlying mathematical relationships and meanings and establish links between mathematical ideas. For example: A child is solving a subtraction problem by taking one ten from tens column. The teacher asks: Why did we rewrite '3' as '13' and why did we change the '2' to '1'.

Linking and applying: These questions focus on the relationship among mathematical ideas and between mathematics and daily life or other subjects. For example: What do we say for half of half kilogram (kg)? How many quarter kgs make 1 kg?

Extending thinking: These questions are aimed at extending an idea so that it can be used in another similar situation. For example: If a pattern is visible in the table of 3, can you see the similar pattern in other tables? Another example could be—If there are six leaves and we arrange them in pairs, nothing is left. Does this happen for all numbers?

Orienting or focusing: These questions help the students to focus on key elements or aspects of a situation in order to enable problem-solving. For example, in the table of 3, we find that odd and even numbers alternate: 3, 6, 9, 12... Is there a similar pattern in other tables?

Generating discussion: These questions help in starting or carrying forward a discussion. For example, is there any other way of doing this? Has somebody done it differently? (Don't worry whether it is correct.) Can we do it with addition instead of subtraction? These are, particularly, helpful in involving learners, who do not participate actively in the class to contribute and comment on ideas being discussed.

Conclusion

Researches on teachers teaching mathematics indicate that the characteristics of questioning determine the extent to which learners are provided with opportunities to participate actively in the teaching-learning process, and construct mathematical meaning. In recent years, one-to-one teaching programmes have come to the fore with a renewed focus to see that all students have a reasonable chance of being successful in school, and to assist children who are at a risk of failure. Better pre-service training in the art of posing classroom questions, along with in-service training to sharpen teachers' questioning skills, have the potential of increasing students' classroom participation and achievement. Increasing the wait-time and the incidence of higher cognitive questions, in particular, have a considerable promise for improving the effectiveness of classroom instruction.

REFERENCES

Almeida, P., H. Pedrosa de Jesus and M. Watts. 2008. Developing a mini-project: Students' questions and learning styles. *The Psychology of Education Review*. Vol. 32. pp. 6–17.

Brophy, J. and T. Good. 1986. Teacher behaviour and student achievement. In M. Wittrock (Ed.), *Handbook of Research on Teaching* (3 Ed., pp. 362–367). MacMillan Publishing, New York.

Brown, G.A. and R. Edmondson. 1985. Asking questions. In E. C. Wragg (Ed.), *Classroom Teaching Skills*. Croom Helm, London. pp. 97–120.

Cazden, C. 1986. Classroom discourse. In M. Wittrock (Ed.), *Handbook of Research of Teaching* (3 Ed., pp. 438–453). MacMillan Publishing, New York.

Chin, C. and J. Osborne. 2008. Students' questions: A potential resource for teaching and learning science. *Studies in Science Education*, pp. 1–39, 44.

Duncan, G.J., et al. 2007. School Readiness and Later Achievement. *Developmental Psychology*. Vol. 43. No. 6. pp. 1428–1446.

Graesser, A. and N.K. Person. 1994. Question asking during tutoring. *American Educational Research Journal*. Vol. 31. pp. 104–137.

Graesser, A. and Olde. 2003. How does one know whether a person understands a device? The quality of the questions the person asks when the device breaks down. *Journal of Educational Psychology*. Vol. 95. pp. 524–536.

Klibanoff, R., et al. 2006. Pre-school children's mathematical knowledge: The effect of teacher "math talk". *Developmental Psychology*. Vol. 42. No. 1. pp. 59–69.

Levin, T. and R. Long, 1981. *Effective Instruction*. Association for Supervision and Curriculum Development. Washington, DC.

Lyons, C., G. Su Pinnell and D. DeFord. 1993. Partners in learning: *Teachers and Children in Reading Recovery*. Teachers College Press, New York.

NCERT. 2010. *Mathematics Teachers' Training Manual for Class* I *and* II. NCERT, New Delhi.

Rowe, M.B. 1986. Wait-time: Slowing down may be a way of speeding up. *Journal of Teacher Education*. Vol. 37. pp. 43–50.

Wright, R.J. November 1994. *Mathematics Recovery: Intervention in the Early Years of Learning*. Presented at the EMT Thirty Years of Excellence Conference. University of Georgia. Athens, G.A.

2

Mathematics that We Know and Use

Anup Kumar Rajput*

Abstract

People, generally, have a wrong notion that mathematics is a difficult subject and believe that it is impossible to enjoy it. It is a fact that math is a part of everyday life. The understanding of mathematics comes very early and naturally in life. In other words, every one can enjoy, engage with, and relax with mathematics. Mathematics is much more — in fact, a lot more than numbers and working algebra formulas. These aspects sharpen the mathematical skills that one may already posses, just as speaking and writing is learned more by language skills. Many subjects and aspects are so deeply integrated with mathematics that it is hard to define it. Mostly in schools, one definition of mathematics prominently learnt is, "Mathematics is the study of quantities and relations through the use of symbols, numbers and rules". This article will help you to appreciate how mathematics is not only a part of you but of animals as well. Secondly, you will see how mathematical problems can be solved faster and with ease. You will also see how to skip the steps entirely and still find accurate or workable solutions to mathematical problems, perhaps without even using a pen and paper. You will, finally, realise that you have known mathematics and that it is an imperceptible part of your daily life.

Introduction

Animals, other than humans, also use and know some mathematical skills, like research studies have shown that crows know how to keep a track of upto 30 persons. Bees can measure angles and lengths. And, almost all animals learn to recognise shapes and sizes. Rabbits need to learn the shape of a flying hawk

* *Professor,* Department of Elementary Education, NCERT

so that they can protect themselves. For animals, mathematics means survival.

Some basic mathematical abilities are probably inborn in all human beings. Almost anyone can tell the difference between two objects, identify the difference in the sizes of two unequal objects, and recognise the difference between a triangle and a circle. But higher level of mathematical abilities requires training. In order to tell the difference between 12 and 13 horses or between a litre of water in a bucket and one kg of edible oil, you must learn special techniques. Mathematics education aims to build inborn abilities, and gradually, take them to higher levels.

Let us now see how one can be sure that crows count. Crows are considered a nuisance in fields because they eat plants. That is why, scarecrows are placed in farms to scare them away, or sometimes, they are hit by shotguns. However, if a crow sees a person with a shotgun, it will not enter the field until the person leaves. The crow must have some recognition of the shape of the shotgun, therefore, a farmer may build a hiding place in the field called 'blind'. Even then, crows are hard to fool. If they see a person entering the blind, they will not pound on to the field until the person leaves.

One farmer thought of an easy solution. Two persons entered the blind, but only one came out. The person, who was left would shoot away the crows when they flew into the field. The crows did not come to the field until the second person had left the blind.

Then, three persons entered the blind and two came out. Even then, the crows were not fooled. Four people going in and three coming out did not fool the crows either. This was the point when everyone became curious about how high the crows could count.

At this point, everyone became curious about how crows kept a record of the number of people. So, the farmer in-charge asked more people to enter the blind. It was not until 30 people entered the blind and 29 came out that the crows were fooled into the field. It meant that the crows had finally 'lost count'.

Counting Numbers

What is counting? Adult human beings can, usually, count up to five objects without using any special technique, and therefore, numbers up to five are called 'perceptual numbers'. A person can tell how many books are there in a stack of four or five without actually counting. Even if a stack contains six or seven books, a person must count in order to tell the exact number. Counting is done by matching each book in the stack with a number name. People learn different number names and rules for combining the names to form numbers in order from one onwards. A person may count the books by saying, "One, two, three, four, five, six, seven..." The person matches each number name with one book in the stack. It tells how many books are there in the stack.

Crows probably 'count' by mental technique that humans use for five or fewer objects. Since crows cannot use language, they have developed the ability to judge larger quantities by sight.

Number name was probably not the first method that humans used to count. Long ago, humans used sets of objects to match things they wanted to count. For example, a shepherd, who wanted to make sure that all sheep were safe for the night, would match each sheep with a pebble and keep the pebbles in a bag. Each night, the shepherd would check to see if there was a sheep for each pebble and a pebble for each sheep. Hence, the sheep were counted In that way, even though no number name was used. The matching process was more important than the use of number name.

Archeologists have found hollow clay balls filled with markers along the trade routes in the Middle East. It is believed that ancient merchants used these balls to tell buyers how many items they had sent. For example, if 17 bars of copper were shipped from one place to another, then a ball containing 17 markers would be shipped, too. When the shipment reached the destination, the buyer would break open the ball, match the markers with the number of copper bars and know if the exact amount had arrived safely.

Eventually, the markers were shown as dents on the outside of the ball, so that people could check the number along the way without breaking the ball open. The clay was baked hard after the dents were made, so that no modification could be done. Dents became the first system of writing numbers. It is also known as the 'cuneiform system'. In fact, people developed ways to write numerals before they developed ways to write words. About five thousand years ago, Babylonians used numerals that looked like the signs as shown in Fig. 2.1

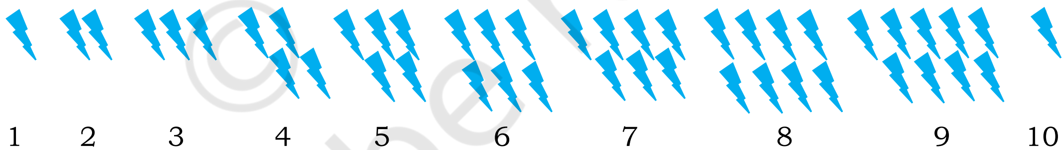


Fig. 2.1: The number symbols used by Babylonians

Measurement

The counting process results in a whole set of numbers—1, 2, 3, 4, 5, and so on and forth. These numbers are commonly called 'counting' or 'natural numbers' as it is believed that no one person or civilisation can be credited for their invention. Counting numbers are the basis of all numbers but they are not enough to solve all mathematical problems that might arise.

For example, a merchant has more than enough copper to make six bars but not enough to make seven. If he wants to ship all the copper, he would need a way to show that he is sending six whole bars and one partial bar. How would he convey this information to the buyer? The answer is to use what today are called 'fractions'.

Fractions are numbers but they are different from counting numbers. If two partial bars of equal size make one whole bar, then each partial is a half of the whole bar. If three partial bars of equal size make one whole, then the size of each partial bar is a third. In each case, a measurement takes place. The merchant is measuring the size of the partial bar in terms of the whole number 1. Fractions, thus, allow the merchant to measure the partial quantity against the whole quantity.

Things may become a bit more complicated for the merchant. Perhaps, the amount of leftover copper he wishes to send will not 'go evenly' into one bar. For example, it will take three partial copper bars to make two (not one) whole bars. The easy solution is to use fraction.

Fraction is a way of showing a relationship between two numbers— the number of parts and that of wholes. If you had any difficulty following the example of the merchant, try this: divide a chocolate bar and give your friend one half. He has half of the whole bar. You split the bar down the middle. Neither one of you is confused because fractions are a natural part of the way you think. You share via fractions.

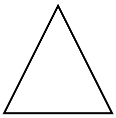
One number in a fraction tells in how many parts of equal sizes the whole bar was divided into. The other numbers tell how many parts are in the piece being measured. Here is a new way of thinking about the merchant's copper bars and using fractions to split the bars mentally.

Can you split a candy bar into halves? Thirds? Fifths? Then, you can use fractions correctly and understand the mathematical concept ratio. A ratio between two quantities is the number of times one contains the other. Since fractions show a ratio between two numbers, mathematicians call fractions 'rational numbers'. You do not need mathematicians to explain this concept, as you have been using it for years.

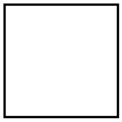
The measurement of any quantity is being done by using counting numbers or fractions in terms of some quantity called 'basic unit'. For example, a piece of length is to be taken as a unit and all lengths, then, can be measured using whole or part of this unit, likewise, all other quantities, such as mass, area and volume or capacity.

Shape

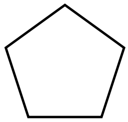
Shape is an important concept in mathematics. Shape can be defined in terms of numbers. Can you think of a shape that has three sides? Four sides?



Triangle



Square



Pentagon

All these shapes are associated with numbers. A figure having three straight lines has to be a triangle. It cannot be a square. Four straight lines of equal length make a square but never a pentagon. The five sides of a pentagon cannot be put into the shape of a triangle. Each figure has its own characteristics, which mathematicians call properties. The properties vary from figure to figure.

Take three sticks and fasten them at the ends (note that the sticks are of equal length). You have just made a triangle. You can try pushing it into different forms but it does not change. It is rigid. Now, add a fourth stick. Push the sticks into different positions. Do you always have a square? No. Sometimes, you have a parallelogram too. What does this mean in terms of numbers? It means that if you have a shape made of three straight lines, you are going to have a triangle no matter how you try to move the sides around. If you have a shape made of four straight lines of equal length, you may have more than one type of a four-sided figure. This difference in properties between the triangle and the four-sided figure is intricately interwoven with numbers 3 and 4.

Measurement enters into shape as well. If all sides of a four-sided figure made of straight lines having the same length, the shape has one set of properties. Else, the shape has another set of properties. The same goes for triangles.

A triangle measuring 3 cm, 4 cm and 5 cm is an important shape in mathematics. Any triangle whose sides have the measurements 3, 4, 5 — no matter what the measurement units are — always makes an angle of the same degree between the three-unit side and the four-unit side. This angle is called the right angle, and thus, the triangle is called the right-angled triangle.

The properties of the right-angled triangle have interested mathematicians for thousands of years, like Bodhayan and Pythagoras.

Patterns

Through ages, people did not have to go to school to see that number and measurement were closely related. They saw that there were patterns in counting and measuring physical objects. For example, a pair of shoes and twin eyes both mean two objects, but no one ever says “a twin of shoes”. In a certain North American Indian language, different number words are used for living things, round things, long times and days. Fiji language uses one word for 10 coconuts and another for 10 boats. These words developed without the basic pattern involved in ‘twoness’, ‘tenness’, ‘hundredness’, or number in general.

Similarly, with the triangular shape, what is important is not what the triangle is made of—just as twoness does not depend on whether the objects are shoes or eyes. In fact, people began to think that a triangle, like a number, was a pattern.

Many conclusions were drawn from this observation. Both number and shape have been dealt with in mathematics because both follow patterns. In other words, mathematics is the study of patterns, and the study of patterns is mathematics.

Logic and Proofs

Mathematics has some fairly obvious patterns. Consider a pattern, such as the following:

$1 + 3 = 3 + 1$ $\qquad$ $11 + 5 = 5 + 11$

$47 + 38 = 38 + 47$ $\qquad$ $332 + 6 = 6 + 332$

You can observe that this pattern holds true for many pairs of counting numbers. But no matter how many pairs of numbers you check, there will be pairs that you have not checked. If you want to be sure that the pattern holds true for all pairs of counting numbers, you must go beyond simply seeing that the pattern is true for a number of pairs.

One of the convincing ways to ensure that the pattern is true for all pairs of counting numbers is to use logic. Logic is also called 'reasoning'. Logic is an argument that as one set of conditions is true, a given result must follow. For example, if you know that

All human beings are mortal.

Suresh is a human being.

Then, you also know that.

Suresh is mortal.

This example is a kind of logical scheme of formal argument called 'syllogism'. But the arguments of logic can be less formal than that. For example, a multiplication problem, such as 4×5, is shown as a set of beads, four rows each with five beads (Fig. 2.2b).

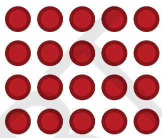


Fig. 2.2 a

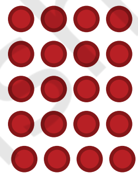


Fig. 2.2 b

You can also show the problem 5×4 with five rows each with four beads (Fig. 2.2a). If you turn the second arrangement, you will find the first one. Obviously, turning the arrangements does not change, therefore, $4\times5 = 5\times4$. The same reasoning will apply to 37 beads, 48 beads, and in fact, to any number of beads in arrangement of rows and columns. This line of thinking is a proof of the fact that for counting numbers, called 'n' and 'm' here, it is always true that

$$n\times m = m \times n$$

It does not matter which two counting numbers 'n' and 'm' are.

Logic is the main tool for finding patterns but is not the same as mathematics. Logic by itself, however, does not go far enough. More than 2000 years ago, during the period of the ancient Greeks, mathematicians had tried to set up perfect rules for logic and math — rules that everyone could agree with. Then, it would be possible to say what really was a proof and what was not. For example, how do you know that turning the arrangement does not change the number of beads? Should turning the arrangement be accepted as a proof — a legitimate way to solve problems in math?

Greeks believed that there were a few simple rules of logic and math that everyone could accept and they called the rules of logic 'axioms' and those of math 'postulates'. This idea helped in establishing the truthfulness of many assertions. For example, when applied to the study of shapes, Greek mathematician Euclid (305–285 B.C.) was able to show that about five axioms and five postulates were enough to prove everything that was known (later, mathematicians improved on his system, but not on the basic idea). This approach to mathematics is called an 'axiomatic system'. As a result of Euclid's success, it became common to think of proof as something that happened only in axiomatic systems. But in reality, early mathematicians proved results in whatever ways they could.

Counting, measurement, shape, patterns, logic and proof — all are parts of math that are easy to think about. The ideas discussed above are common and used in our daily life. They became a part of our life, language and tool to solve problems. Our thinking patterns have acquired these ideas in a way that lead to most of our conclusions. In other words, mathematics in our mind is what we use and dwell on further.

REFERENCES

ARISTOTLE. *Inductive Reasoning.*

3

Learning from Errors in Numbers and Number Operations in Early School Mathematics

Anchal Arora*

Abstract

Errors are an inevitable part of the learning process. This article begins with the need to re-conceptualise errors in the learning of mathematics from obstacles or hindrance to insights to learners' thinking process, and opportunities for learning. The latter section of the article focuses on error analysis with reference to the concept of numbers in early school mathematics. It discusses what error analysis means and how it can play an important role in integrating assessment with learning, as well as, help shift focus from right or wrong answers to a broader meaning of learning in mathematics.

KEYWORDS : Error analysis, numbers, number operations, assessment for learning

The National Curriculum Framework (NCF)–2005 bases itself on the principle of mathematics for all. According to this principle, every learner is seen as capable of learning mathematics and all should experience the joy of learning the subject. In conjunction to this, it recommends the assessment to be continuous and comprehensive in nature. Continuous, here, means that assessment should become an ongoing process. The need is to integrate assessment with the daily teaching-learning process, focusing on students' thinking and learning. Comprehensive means to cover a wide range of aspects of learning, like attitudes and skills, (for example, creativity and ability to communicate clearly and analyse) and not simply content knowledge. However, the teaching-learning of mathematics is burdened by approaches focusing on algorithm and one correct answer. Under such an approach, often a learner is evaluated on the basis of his/her ability to get the correct answer. An incorrect response symbolises the lack of understanding. A learner who experiences failure in geting the correct

* *Consultant*, Early School Mathematics Programme, DEE, NCERT

answer for some time is vulnerable to be labelled as unintelligent or lacking ability (Boaler, 2013). A plethora of research literature in mathematics education argues for a shift in teaching-learning and assessment practices, focusing more on learners' thinking and responses (Cooper and Dunne, 2000; Lerman and Zevenbergen, 2004; Ryan and William, 2007; Cockburn, 1999). The role of assessment in the development of mindsets and learners' identity is considered crucial. The first section discusses what are errors with reference to number sense in early school mathematics, i.e., Class I and II. The second section explores the scope and need of error analysis as an important tool integrating assessment with learning.

What are Errors?

Learners' alternate responses in given tasks can be classified into two categories. Firstly, like any human failure, learners' alternate response in the given tasks can be a consequence of slips (Ryan and William, 2007). These slips actually have a 'chance element'. These can be termed as 'mistakes'. These slips or 'chance elements' or 'mistakes' do not have any developmental or conceptual explanation. Researchers have found that factors, like misreading and quickly jumping to an answer or conclusions (Sweller, 1994) are reasons for such mistakes or slips. Secondly, the lack of performance can be traced to have a conceptual or developmental basis. A learner's alternate response in a given task can be the consequence of partial, alternate or misconceived conceptual understanding of a mathematical concept (Ryan and William, 2007; Cockburn, 1999). The differentiation between an error and a mistake in a given alternate response is difficult to make. For the convenience of categorisation, if a learner is able to self-correct the response, it can be put in the category of a mistake, else it can be considered as an error. The next section explores the nature and probable reasons for committing errors with relation to number operations.

Errors in Early School Mathematics— An Elaboration through Numbers and Number Operations

The concept of numbers starts developing in children at an early age. Research has brought forth the informal knowledge that learners develop about numbers at an early age (Bryant, 1997; Ginsberg, Choi, Lopez, Netley and Chao-Yuan, 1997). Number concept is one of the core components of school mathematics (Schoenfeld, 2007, and Kilpatrick, 2001). Often numbers are taken as a simple and obvious concept to be learned. But literature describes there are various skills and sub-concepts, which learners may need to learn about numbers with understanding (Cockburn, 1999; Ryan and Williams, 2007). For instance, the simple looking counting process involves pre-number concepts, like one-to-one correspondence, seriation, classification and a knowledge of number names in a correct order (NCERT, 2010). The complexities involved in the learning of numbers and number operations, along with various other factors, like teaching-learning process, language and previous exposure of learners, make errors an inevitable part of the learning process. Errors can be classified on the basis of different criteria. The various sub-concepts involved in numbers and number operations can help categorise the errors learners commit in early school mathematics. For instance, errors in addition and subtraction can be due to the following:

- the lack of understanding of regrouping;
- confusion of 1s and 10s in carrying and writing;
- forgetting to carry 10s and 100s;
- forgetting to regroup when subtracting 10s and 100s;
- regrouping when not required;
- inappropriate use of operation (addition instead of subtraction or vice versa);
- the lack of knowledge of basic number facts;
- the lack of knowledge about the concept of zero;
- over-genralisation: bringing the concept or rule learned for one sub-concept or concept to other where it does not fit; and
- prototyping: generalisation of a concept or sub-concept to only familiar or commonly used examples or situations.

(Adapted from Ryan and Williams, 2007)

With the teaching-learning process in focus, the three major factors contributing to errors in number operations are discussed below. Firstly, teaching or following thumb rules contributes to errors in number operations. By thumb rules, one refers to the shortcuts that teachers tell learners or learners follow in order to arrive at a solution quickly. These thumb rules restrain learners' engagement with a concept, i.e., logic or meaning of the concept. For instance, as we have discussed in the example mentioned in the section on error analysis, there is a possibility that the teacher used thumb rules or the child remembered the rules. In number operations, if on adding two 'ones' digits we get a two digit, then one of the digits needs to be taken to the other place. But the child fails to understand the logic and takes over any of the two digits to the next place.

Secondly, errors can be due to erroneous teaching and learning, i.e., content or unintended aspects. For example, during the teaching of area and perimeter, it was observed that a teacher throughout the unit used cm and km as units of area. And, interestingly, when learners from that class were interviewed on problems related to area, they gave responses for units of area in cm/m/km (Arora, 2011). Here, it can be said that erroneous teaching by the teacher might have caused errors made by the learners.

Thirdly, errors can be due to the usage of examples, which may lead to over-generalisation or prototyping. For instance, in case of number operations, the keyword 'more' is generalised for addition. Consider a problem situation — Fozia sells flowers to passersby on a red light in Delhi. She sells the flowers in bunches of 12. She has nine flowers. How many more flowers does she need to make a bunch of 12? Some children in such a situation may add 9 and 12 and give 21 as their response, instead of subtracting 9 from 12. This can be the case of over-generalisation or prototyping, where the children may have responded due to the usage of the word 'more' in the problem.

The above mentioned classification and reasons for errors are suggestive and not comprehensive or decisive in nature.

What is Error Analysis?

The following work illustrates an error made by a Class II learner in the concept of addition with regrouping.

Here, the child was unable to get the correct answer for some sums. But interestingly, there was a pattern in which the child gave the responses. It would be unfair to consider that the child had no idea of addition. He probably had some idea of addition with single digits, i.e., addition without regrouping. He also knew that if the sum of two digits at 'ones' place resulted in a two-digit number, then one digit had to be taken to the other place. But probably the child did not have an idea of place value. Also, he was unable to reason out which digit should be taken to the next place for regrouping and why? This error analysis is probabilistic in nature, given the lack of evidence.

To reach a certain informed understanding of the child's thinking process, it would be required to give more focused tasks or sums. It is even more important to talk and let the child articulate what he is doing and why. This would be crucial in understanding the problem area and what needs to be done to address it.

Why Error Analysis?

The following learners' responses can be useful for various reasons. Firstly, it can be a useful tool for teachers, who can employ assessment in a continuous and comprehensive manner. Secondly, it can help break the conventional notion of teaching-learning of mathematics, where mastery to reach the correct answer is a dominant practice. It instead can help promote a discursive classroom, where the process of learning becomes as important as mastery over a concept or procedure. In the process, learners are encouraged to think mathematicly by having mathematical discussions, logical arguments and develop an in-depth conceptual and procedural fluency (Ryan and Willams, 2007).

Integrating Assessment with Learning

Grigorenko and Sternberg (1998) argue for a dynamic form of assessment, which does not evaluate learners but focuses on their thinking processes and helps in understanding their current abilities to support the development of their potential. Thus, it is argued that dynamic assessment does not restrict evaluation to the final outcome but gives access to learners' thinking process and potential to learn simultaneously (Lidz, 1987, 1991, cited in Sternberg, 2001; Grigorenko and Sternberg, 1998, Shephard, 2000). Error analysis can be one of the key components in assessment for learning in mathematics (Hodgen and Askew, 2010).

As discussed above, error analysis can help facilitators get an insight into the learners' thinking process and complement assessment for learning. It emphasises on conceptual gaps and turns them into opportunities for teaching and learning. The teaching-learning processes and assessment practices, which utilise errors as opportunities of learning, can help create a positive learning environment. It helps avoid the labeling of learners as poor, weak, or unintelligent in mathematics. Error analysis helps teachers understand what a learner may know and needs to know instead of labeling him/her for what he/she does not know. Teachers may start considering errors as natural steps towards learning. This may help in moving away from labeling learners as intelligent or unintelligent, and provide qualitative feedback, which supports further learning. Qualitative feedback, which is elaborative in nature and focuses on effort and learners' thinking, can help promote the develpoment of growth mindsets among leaders.

Learning in Mathematics: Moving beyond Right or Wrong

A vast literature points to the fear and anxiety many learners associate with school mathematics. It also points out that learners' views about themselves in relation to mathematics is found to be influenced by their marks, ability to give correct responses, and how teachers, peers and parents rated them in the subject (Boaler, Wiliam and Brown, 2000; Reay and Wiliam, 2009). For instance, Boaler, Wiliam and Brown (2000) found that teachers somewhere considered getting to the correct answer quickly without committing mistakes as a marker of one's ability to solve mathematical problems.

On the other hand, an emergent body of literature points out how mistakes should be seen as a stepping stone to learning (Dweck, 2012, cited in Boaler, 2013). Errors are a natural and an inevitable part of learning. Instead of focusing on what the learner does not know, error analysis helps to understand what he/she knows and what he/she needs to know. It helps in designing the teaching-learning processes, which can lead to the development of potential abilities in learners (Hodgen and Askew, 2010). Errors signify the active involvement of learners in the learning process. They help in shifting the focus on the process of learning, learners' effort and thinking process than merely seeing the child's work in terms of correct or incorrect responses. For instance, in teaching a concept, a special session on learners' alternate responses can also become a part of the teaching process. An especially designed worksheet, containing learners' alternate responses, can be given to learners in groups to discuss. They can be asked to decode how and why some child gave such a response, and what was his/her logic. Such error eliciting tasks would help learners gain an in-depth understanding of the concept. Also, it encourages in establishing a motivated learning environment, where the learners are not afraid to make mistakes and see them as an inevitable part of the learning process. This, consequently, can help develop a better self-esteem among learners, positive attitude towards learning mathematics and growth mindset towards learning the subject (Dweck, 2006).

For error analysis to be evidence-based, it is important that multiple sources are used to collect data. Based on the initial data collected, a teacher should use one-to-one task-based interviews to understand the thinking process of the learners behind such responses. These interviews can consist of varied activities, like worksheets, or working with concrete learning material, or oral problem situations. These interactions can be taken as a means to encourage the learners to articulate their thinking and reasoning. This would help find the patterns or logic behind the learners' responses. For instance, the section on error analysis mentioned above attempts to illustrate the initial process of error identification and analysis, using a learner's response to an addition of two-digit sum, requiring regrouping. But this error analysis cannot be considered complete unless the learner is given an opportunity to articulate his/her thinking and the logic behind it.

Tools, like in-depth observations of teaching-learning process, assessment practices, like study of learners' work, one-to-one interviews and focused group tasks can be used to recognise the common errors learners make while learning various concepts in school mathematics. Based on the data collected, the identified errors in a particular concept can be analysed and categorised through thematic analysis. A classroom intervention can be designed based on themes that emerge from the thematic analysis of the errors that learners make. The intervention can be undertaken either with a purposively selected group of learners facing challenges or, generally, with all learners depending on the need and context of the classroom. For instance, in the above case, where the learner was making errors in addition to regrouping, the intervention can focus on both number concept, as well as, place value. Skills, like estimation and checking, can help a child identify the inappropriate procedure he/she is using to add. Varied forms of activities, such as open-ended tasks (Boaler, 2013; Sullivan and Lilburn, 1997), tasks using teaching-learning material, like arrow cards, play money, bead strings, board and card games can help the child build an in-depth understanding of the concept.

Conclusion

As literature suggests, there is a need to pay a greater attention to assessment practices, which contribute to students' learning (Stiggins, 2002; Black, 2004; Shephard, 2000; Ruthven, 1994). However, workable ideas, which can help integrate assessment with learning still remain a challenge. Error analysis can provide a tool kit to design strategies for implementing assessment for learning, in general, and mathematics, in particular. As discussed in the article, error analysis of responses provides a window to learners' thinking and learning process. It helps in understanding what a learner may know and needs to know instead of labeling him/her for what he/she does not know. It may help learners, as well as, teachers to appreciate the incremental nature of intelligence and abilities, along with the development of a growth mindset, where efforts and curiosity to learn and accept challenging tasks become a part of the learning process. But prior to this, there is a need to accept errors as a natural and an inevitable aspect of the learning process. It is only then that errors can be re-conceptualised from hindrance or obstacles to insights for learning (Ryan and William, 2007) and error analysis can become part of teaching-learning and assessment practices.

REFERENCES

Anderson, R. 2007. Being a Mathematics Learner: Four Faces of Identity. *The Mathematics Educator*. Vol. 17. No.1. pp. 7–14.

Arora, A. 2011. Analyse how the New NCERT Textbooks based on NCF–2005, are being implemented and their impact on the classroom teaching-learning process. Unpublished dissertation submitted under the Masters Programme to Tata Institute of Social Sciences, Mumbai.

Black, P. 2004. Working inside the Black Box: Assessment for Learning in the Classroom. Granada Learning. *Phi Delta Kappan.* Sage Publications, Virginia. Vol. 84. No. 1. pp. 9 –21.

BLACKWELL, L. S., K. H. TRZESNIEWSKI AND C. S. DWECK. 2007. Implicit Theories of Intelligence Predict Achievement across an Adolescent Transition: A Longitudinal Study and an Intervention. *Child Development*. Vol. 78. No. 1. pp. 246–263.

BOALER, J. 2013. Ability and Mathematics: The Mindset Revolution that is reshaping Education. FORUM. Vol. 55. No. 1. pp. 143–152.

BOALER, J., D. WILIAM AND M. BROWN. 2000. Students' experiences of ability grouping—disaffection, polarisation and the construction of failure. *British Educational Research Journal*. Vol. 26. No. 5. pp. 631–648.

BOALER, J. AND M. STAPLES. 2008. Creating Mathematical Futures through an Equitable Teaching Approach: The Case of Railside School. *The Teachers College Record*. Vol. 110. No. 3. pp. 608–645.

BROWN, M., P. BROWN AND T. BIBBY. 2008. 'I would rather die': Reasons given by 16-year-olds for Not continuing their Study of Mathematics. *Research in Mathematics Education*. Vol. 10. No. 1. pp. 3–18.

BRYANT, P. 1997. Mathematical Understanding in Nursery School Years. In T. Nunes and P. Bryant (Eds.). *Learning and Teaching Mathematics: An International Perspective*. Psychology Press, UK. pp. 52–67.

COCKBURN, A.D. 1999. *Teaching Mathematics with Insight: The Identification, Diagnosis and Remediation of Young Children's Mathematical Errors*. Routledge Falmer, London and New York.

COOPER, B. AND M. DUNNE. 2000. *Assessing Children's Mathematical Knowledge: Social Class, Sex and Problem Solving*. Open University Press, Philadelphia, Buckingham.

DWECK, C.S. 2006. *Mindset: A New Psychology of Success*. Random House, New York.

GINSBURG, H.P., Y.E. CHOI, L.S. LOPEZ, R. NETLEY AND C. CHAO-YUAN. 1997. Happy Birthday to You: Early Mathematical Thinking of Asian, South American, and U.S. Children. In T. Nunes, and P. Bryant (Eds.), *Learning and Teaching Mathematics: An International Perspective*. Psychology Press, UK. pp. 163–207.

GRIGORENKO, E.L. AND R.J. STERNBERG. 1998. Dynamic Testing. *Psychological Bulletin*. Vol. 124. No. 1. pp. 75–111.

HODGEN, J. AND M. ASKEW. 2010. Assessment for Learning: What is All the Fuss about? In I. Thompson (Ed.), *Issues in Teaching Numeracy at Primary Schools*. Open University Press, Berkshire and New York. pp. 133–145.

KILPATRICK, J. 2001. Understanding Mathematical Literacy: The Contribution of Research. *Educational Studies in Mathematics*. Vol. 47. No. 1. pp. 101–116.

LERMAN, S. AND R. ZEVENBERGEN. 2004. The Socio-political Context of the Mathematics Classroom: Using Bernstein's Theoretical Framework to Understand Classroom Communications. In P. Valero and R. Zevenbergen (Eds.), *Researching the Socio-Political Dimensions of Mathematics Education: Issues of Power in Theory and Methodology*. Kulwer Academic Publishers, New York, Dordrecht, London, Moscow. pp. 27–42.

National Council for Educational Research and Training. 2005. *National Curriculum Framework*. NCERT, New Delhi, India.

———. 2010. Mathematics Teachers' Training Manual for Classes I and II. NCERT, New Delhi, India.

Reay, D. and D. William. 1999. 'I'll be Nothing': Structure, Agency and the Construction of Identity through Assessment. *British Educational Research Journal*. Vol. 25. No. 3. pp. 343–345.

Ruthven, K. 1994. Better Judgement: Rethinking Assessment in Mathematics Education. *Educational Studies in Mathematics*. Vol. 27. No. 4. pp. 433–450.

Ryan, J. and J. Williams. 2007. *Children's Mathematics 4–15: Learning from Errors and Misconceptions*. Open University Press, England.

Schoenfeld, A.H. 2007. What is mathematical proficiency and how it can be assessed? In A.H. Schoenfeld (Ed.). *Assessing Mathematical Proficiency*. Cambridge University Press, Cambridge. pp. 59–73.

Shephard, L.A. 2000. The role of assessment in a learning culture. *Educational Researcher*. pp. 4–14.

Sternberg, R.J. 2001. Giftedness as a Developing Expertise: A Theory of Interface between High Abilities and Achieved Excellence. *High Ability Studies*. Vol. 12. No. 2. pp. 159–179.

Stiggins, R.J. 2002. Assessment crisis: The absence of assessment for learning. *Phi Delta Kappan*. Vol. 83. No. 10. pp. 758–765.

Sullivan, P. and P. Lilburn. 1997. *Open-ended Maths Activities: Using 'Good' Questions to Enhance Learning*. Oxford University Press, Melbourne.

Sullivan, P., S. Tobias and A. McDonough. 2006. Perhaps the Decision of Some Students not to Engage in Learning Mathematics in School is Deliberate. *Educational Studies in Mathematics*. Vol. 62. pp. 81–99.

Sweller, J. 1994. Cognitive Load Theory, Learning Difficulty and Instructional Design. *Learning and Instruction*. Vol. 4. No. 4. pp. 295–312.

4

Understanding Numbers — Concepts and Some Misconceptions

Poonam Pant*
Vyomesh Pant**

Abstract

Interactions with students and teachers in NDMC schools reveal that there are some problems in the understanding of numbers during teaching-learning in classrooms. This paper tries to address the importance of numbers in mathematics, as well as, in our daily life. Though numbers are an essential part of our day-to-day life and mathematics cannot exist without numbers, students and teachers still have misconceptions about numbers. This paper attempts to discuss with the help of examples some of the problems faced by primary level children and teachers.

Introduction

"Without Mathematics, there is nothing you can do. Everything around you is mathematics, everything around you is numbers."

— Shakuntala Devi

This quote of Shakuntala Devi presents a picture about the importance of mathematics and numbers in our life. Numbers are an essential part of mathematics and a large portion of the subject is developed around numbers.

Can we imagine our life without numbers? If there is no number in our life, then how will we decide the date and time? How will we determine the route number of a school bus or bus to our workplace? How will we decide our age? How will we distinguish and identify a car without a number plate? What will be the form of the currency? How will money be transacted? Think of life without numbers. You will find that life is certainly impossible without numbers.

* *Academic Coordinator*, NSES, Department of Education, New Delhi Municipal Council, New Delhi
** *Research Scholar* in Mathematics

When we talk of numbers, we, generally, think about 1, 2, 3, 4... In mathematics, there are so many numbers — whole numbers, natural numbers, integers, rational numbers, irrational numbers, real numbers, complex numbers, etc. But when we talk of numbers, the general perception that comes in the mind of a common man is 1, 2, 3, 4..., i.e., numbers, which are generally used for counting. People, while talking of numbers, seldom think about rational, irrational or complex numbers. One of the reasons behind this may be that everyone is not skilled enough to think about these. But what about people who have the knowledge of all types of numbers but still think like a layman when asked about the same.

Another important point that we would like to emphasise here is that when asked about numbers we normally think of natural numbers. Most of us remain confined to positive numbers and some of us may think of negative numbers. But what about 'zero'? How many of us think about 'zero' at the first instance when we think of a number? Take a sample of 10 people from different age groups, educational and socio-economic backgrounds, and ask them to think about a single-digit number and write it down. You will hardly find that anyone has written 'zero'. What can be concluded now? Is 'zero' not a number, or is there a misconception about the status of 'zero' — whether 'zero' is to be treated as a number or not?

To bring more clarity in the aspect which we want to discuss here, take an example — class teachers of a primary school were given the task of noting down the birthday month of each student in their respective classes and prepare a table with the name of the month in one column and the number of students whose birthdays fall in that month in another column. Suppose, there are two students in a class, whose birthdays fall in January. Then, write 2 against January, and so on. If no child's birthday falls in a month, write '0' against it. A sample of the table prepared by a teacher may be as follows:

S. No.	*Name of the month*	*No. of students having birthdays in a month*
1.	January	02
2.	February	05
3.	March	04
4.	April	04
5.	May	00
6.	June	01
7.	July	03
8.	August	02
9.	September	04
10.	October	02
11.	November	03
12.	December	05

In almost every class, there was a '0' in the list, meaning that there were some months, in which no child was born in a class. The teachers are then asked to tell the month having the minimum number of birthdays. Most of the teachers got confused. They were not sure whether a month having '0' birthday would be the answer. For example, the teacher, who prepared the table (see p.28), was confused whether the month having the least number of birthdays would be May or June. The teacher thought that having '0' birthday in a month meant having no birthday in that month. Therefore, the month having the least number of birthdays would be June, in which only one child's birthday fell. On the other hand, '0' it must be noted that '0' is a number. Therefore, the month having the least number of birthdays should be May. Does '0' actually mean 'nothing' or is it something different from 'nothing?

Take another example — ask a simple question to students. How many numbers are less than 10? Some would say 9, counting the numbers from 1 to 9. Some would say 10, including '0' as one of the numbers. Some may delve deeper and say infinity, including the negative numbers –1, –2, –3... as well. While talking about numbers, we confine ourselves to integers and do not bother about other numbers.

It is natural to ask why there is confusion or variation in our perception about '0' and about the negative numbers, or broadly speaking, why we do not have a clear perception of the entire number system. To remove ambiguity in the perception of numbers, we must understand that numbers are used in different forms. The perception or understanding of numbers may be different with change in its form. There are three forms in which a number can be used — cardinal, ordinal and nominal. Cardinal numbers show quantity and are also known as 'counting numbers'. Cardinal numbers tell 'how many', for example, five children, four computers, nine players, etc. Ordinal numbers show the order of things in a set, for example, first, second, third... Ordinal number shows only the position or rank and does not indicate quantity, for example, third child from the left in the first row, second largest country, etc. Nominal number is used to name something. Nominal number is neither used for counting, nor for indicating a rank or position. It is used to name or identify something, for example, a player wearing jersey number 99, a car having number 0623, an office in an area having postal code 110001, etc.

Consider the following sentences:

1. Route number 6 bus is my school bus.
2. There are six buses in this route for my school.
3. The sixth bus from the right is my school bus.

Here '6' has been used in all the above sentences to denote a bus. In sentence 1, '6' is used as a mark of identification for the school bus to distinguish it from others. Thus, 6 is used here in the nominal form. In sentence 2, '6' is used as a cardinal number because it indicates the number or quantity of the buses. In sentence 3, '6' indicates the position of the bus and is used as an ordinal number. Numbers are used as labels for identifying things (nominal aspect), putting things in order (ordinal aspect) and as indications of how many are there in a set of things (cardinal aspect) [Haylock and Cockburn, 2003].

Haylock has pointed out that some aspects related to numbers cannot be understood, if we think of them in the cardinal sense as a set of things. We have to establish a connection with numbers used in the ordinal sense, as labels for putting things in order. For example, if we talk about a bus having route number 6, the number is mentioned only to identify it from other buses. It is not the case that the bus will follow immediately after the route number 5 bus. In the example given above, when the teacher was asked to point out the month having the least number of birthdays, s/he was perplexed in giving a reply because s/he might have interpreted the situation in two ways — '0' birthday in a month practically means no birthday in that month. Therefore, May cannot be said as the month having the least number of birthdays as no birthday fell in that month. But from the aspect that '0' itself is a number, in fact it is the least number of the set of numbers formed in the example, the answer should be May.

Haylock has explained the difference between cardinal and ordinal numbers. For further reading on the subject, we recommend Haylock and Cockburn. If one is able to understand the difference between cardinal, ordinal and nominal forms of numbers, there will be no problem in describing the numbers and their use.

Infact, '0' is a 'well-decorated' number of the 'number system'. We all know that symmetry has an important place in the study of science and mathematics. Be it the field of architecture or drawing and painting or interior decoration, symmetry gives elegance and perfection. For example, take body parts that are in pair, such as ears, eyes, eyebrows and to some extent nose (pair of nostrils). Any asymmetry in these pairs may lead to ugly look, whereas, symmetry makes our look perfect. For people who love symmetry, '0' is a 'well-decorated' number in the sense that it is the point of symmetry of the Real Line. It occupies the central place Real Line and the position of numbers is the same on both of the sides of '0'. The Number Line starts from '0'. At one side of it positive numbers are placed, whereas, the negative numbers are placed on the other side.

In our daily life also '0' is given a special treatment — '0' floor means the ground floor. There is a section in our society, which likes to live on the ground floor. Ground floor of a multi-storeyed mall or store is normally different from the other floors. It is well-maintained, attractive than the other floors, displays pictures of items available in the store and their price range, provides a detail about the store, offers for customers, etc.

Despite all this, '0' has been getting secondary treatment. Most of us are not ready to treat '0' as a number. Our perception about '0' is not mathematical. Not only '0' but the negative numbers, too, are not understood well by most of us.

Haylock emphasises that the ordinal aspect of numbers may be introduced to students in the primary classes itself, so that the possibility of getting confused in later years is reduced. If I have five chocolates and I distribute these among five children, how many chocolates am I left with? This is the most common way to introduce the concept of '0' to children in primary classes. However, if '0' is introduced like this, how will one introduce negative numbers? What is the meaning of –5, how will you explain it? One of the practical approaches to introduce negative numbers is to take your child to a lift or an elevator. The ground floor is denoted by '0', the basement (upper ground floor) and lower basement (lower ground floor) are denoted by –1 and –2, respectively, and the first, second, third... floors are denoted as 1, 2, 3... in the control panel of the lift. Here, the child can learn the concept of '0' and negative numbers in a practical way. The use of Number Line and the concept of negative temperature may be useful in helping them understand the concept of negative numbers.

Number sense describes the intuitive idea about number understanding and its use in different forms, and one's ability to differentiate between these forms. According to Gersten and Chard, number 'sense' means students should have a sense of what numbers mean. They must understand their relationship with each other and be able to perform mental math by understanding symbolic representations and use those numbers in real life situations. Advocating the relevance of 'number sense', Carlyle and Mercado state in their book that it is important as it encourages students to think flerxibly and promotes confidence with numbers — they make friends with numbers.

The National Council of Teachers in 1989 identified the following five components that characterise number sense:

- number meaning
- number relationships
- number magnitude
- operations involving numbers and referents for number
- referents for numbers and quantities

Hence, numbers are an essential part of our life and we cannot imagine life without numbers. Similarly, mathematics also cannot survive without numbers.

REFERENCES

BURNS, MARILYN. 1997. How I boost my students' number sense. *Instructor Magazine* (April). pp. 49–54.

CARLYLE, ANN AND BRENDA MERCADO. 2012. *Teaching Pre-school and Kindergarten Math: A Multimedia Professional Learning Resource*, Math Solutions, Sausalitolito, CA. 94965.

HAYLOCK, DEREK. 2010. *Mathematics explained for Primary Teachers* (4th Ed). SAGE Publications, London.

HAYLOCK, DEREK AND ANNE D. COCKBURN. 2003. *Understanding Mathematics in the Lower Primary Years: A Guide for Teachers of Children.* SAGE Publications, London. pp. 3–8.

5

Insights into Math Word Problems: Developing Learning Cycle for solving Word Problems

Arushi Kapoor*

Abstract

Word problems in mathematics are real-time problem situations, which can be solved using mathematical concepts. Inclusion of word problems in school mathematics curriculum aims to inculcate problem-solving skills among children. Word problems have always caught the attention of researchers because children face many learning challenges while working on word problems. This paper discusses the outcome of an empirical study conducted on fourth and fifth grade students to find out the learning difficulties they face and learning patterns they follow while working on word problems. George Polya's classical work (1945) on problem-solving and Anne Newman's seminal work (1977) on the working of word problems were used to develop conceptual framework for the study. An in-depth analysis was done to develop insights into the processes adopted by students at various levels of solving word problems. Findings of the study were used to construct suggestive learning framework for solving word problems.

Introduction

Word problem is defined as, "A mathematical problem that is stated in words rather than in symbols or as an equation" (*Mathematics Thesaurus*). Majority of mathematics topics contain word problems. The aim of word problems is to connect a mathematical concept with real life situation. Word problems act as linkages between concept and application. Mathematical word problems are focused to build the aptitude of problem-solving in children. School mathematics of the twenty-first

* *Assistant Professor*, University of Delhi, New Delhi

century is viewed by educators as a subject, which should engage a learner in problem-solving and reasoning. It should also foster understanding and develop the learner's critical and analytical skills. Instruction should not be limited to plain mastery of algorithms or development of certain mathematical skills. It should involve learners in investigation through "exploring, conjecturing, examining and testing" (National Council of Teachers of Mathematics, 1990, p. 95). Successful problem-solving involves coordinating with previous experiences and knowledge to generate new representations and related patterns. In school, mathematics is the domain, which formally addresses problem-solving as a skill. Considering that it is an ability of use in one's life, techniques and approaches learnt in school have immense value. Mathematics also provides an opportunity to make interesting problems (National Curriculum Framework, 2005). It is experienced, particularly, in Indian schools that mathematical problem-solving is a major concern in a student's school life due to various reasons, such as lack of comprehension of the problem posed, strategic knowledge, domain-specific knowledge or experience in defining problems, inability to translate a problem into a mathematical form, tendency to rush towards a solution before the problem has been clearly defined. Therefore, word problems have always caught the attention of researchers to effectively blend problem-solving skills in children.

George Polya (13 December 1887– 7 September 1985) is known as 'Father of Problem Solving'. He was a Hungarian mathematician. He worked as a professor of mathematics from 1914 to 1940 at ETH Zürich, and from 1940 to 1953 at the Stanford University. He made fundamental contributions to combinatory, number theory, numerical analysis and probability theory. He is also noted for his work in heuristics and mathematics education. George Polya's (1945) classical work has been published in his book *How to solve it*, Princeton University Press, 2004. It throws an insight into problem-solving and parameters involved in dealing with word problems. In this book, he identifies four basic parameters of problem-solving.

Table 1: Polya's four parameters for problem-solving

S.No.	*Parameter*	*Meaning*
1.	Understand the problem	Understanding all words used in stating the problem — what is being asked to find or show, restating the problem in own words, and illustrating the problem diagrammatically are enough to find a solution.
2.	Devise a plan	There are many ways to solve problems. Choosing an appropriate strategy to solve the problems is one such way.

3.	Carry out the plan	Persist with the plan that has been chosen. If it does not work, discard it and choose another.
4.	Look back	Take time to reflect on what has been done, what worked and what did not. Doing this will enable you to ascertain what strategy to use in order to solve future problems.

Inspired by the classical work of Polya, Australian educator Anne Newman (1977) suggested five significant parameters to help determine where errors may occur in students' attempts to solve written problems. She developed an 'Error Analysis Model' to classify errors and identified a sequence of steps. This model, known as 'Newman's Error Analysis (NEA) Model', has been used by many researchers to study word problems.

The parameters according to the Newman's Model are given in Table 2.

According to NCERT textbooks, word problems start at the upper primary grade (3^{rd} grade onwards), which demand command over language, transformation, abstract concepts and fundamental knowledge to enable children to handle word problems effectively.

This empirical study dealt with the word problem-solving abilities of fourth and fifth graders. The study was explanatory in nature. An in-depth analysis was conducted to develop insights into the processes adopted by students at various levels of solving word problems. Findings of the study were used to construct suggestive learning framework for solving word problems.

Objectives of the Study

1. To find out which area of word problem students find the most difficult with reference to Newman's approach
2. To find the difference between the problem-solving approach

Table 2: Newman's five parameters of Error Analysis Model

S.No.	*Parameter*	*Meaning*
1.	Reading	Reading the problem
2.	Comprehension	Comprehending what is read
3.	Transformation	Carrying out a transformation from words of the problem to selection of an appropriate mathematical strategy
4.	Process	Applying the process skills as demanded by the strategy selected
5.	Encoding	Encoding the answer in an acceptable written form

of students of government and private schools with reference to Newman's approach

3. To find out the difference between the problem-solving approach of girls and boys with reference to Newman's approach
4. To construct suggestive learning framework for solving word problems

Methodology

It was a qualitative study, wherein children of fourth and fifth grades were surveyed to understand their thought processes while working with word problems and an in-depth analysis was conducted with reference to the Newman's Error Analysis Model using a 3–point rating scale.

Sample

Ten (five boys and five girls) government school students and 10 (five boys and five girls) private school students of Classes IV and V were selected as sample.

Five word problems were constructed after consulting the NCERT syllabus based on many criteria dealing with everyday life. The problems required fundamental knowledge, clubbing of one or more mathematical concepts, etc. Word problems were translated for Hindi-medium students. The conceptual framework of word problems is given in Table 3.

Due permission was taken by the school authorities and appropriate time was given to the students to solve the word problems. They were also provided with small clues as and when required.

Tool: A 3–point rating scale based on Newman's five steps was developed to assess the students' process of solving word problems.

Analysis

An in-depth analysis was conducted on the basis of Newman's Problem-solving Model to develop insights into the processes adopted by students at various levels of solving word problems. The analysis was mainly qualitative in nature but it was also quantitative in order to support the study.

Findings and Analysis of the Study

The students were marked on each Newman's parameter on the basis

Table 3: Conceptual framework of word problems

Word problem	*Underlying concepts*
Problem 1	Concept of calender, arithmetic operations
Problem 2	Concept of time or duration, arithmetic operations
Problem 3	Concept of money, arithmetic operations
Problem 4	Measurement of mass, money, arithmetic operations, estimation
Problem 5	Concept of temperature, arithmetic operations, estimation

of their effectiveness in dealing with word problems.

Findings

A 3–point rating scale was developed to judge all five parameters of Newman's Error Analysis Model, these are — reading the problems, comprehension of the problems, transformation of the problems, processing of the problems, and then, encoding the problems.

Table 4: 3–point rating scale used in the study

S.No.	*Effectiveness*	*Score*
1.	Was not able to do at all	0
2.	With errors	1
3.	Correctly	2

The maximum score for each parameter was 10 (two for each word problem). The minimum score for each parameter was 0 (0 for each word problem).

Table 5: Scoring of the sample

NEWMAN'S PARAMETERS Total score in all five word problems					
Students	*Reading*	*Comprehension*	*Transformation*	*Process*	*Encoding*
GM 1	9	7	6	3	0
GM 2	9	9	6	6	2
GM 3	9	9	8	5	1
GM 4	5	3	2	2	0
GM 5	8	8	5	1	0
GF 1	8	4	2	0	0
GF 2	7	6	2	1	0
GF 3	8	7	5	0	0
GF 4	9	8	5	3	2
GF 5	10	0	0	0	0
PM 1	9	7	5	6	6
PM 2	9	6	5	4	4
PM 3	10	6	5	4	5
PM 4	10	10	7	7	6
PM 5	10	10	10	8	8
PF 1	10	7	9	8	7
PF 2	10	8	9	8	7
PF 3	9	8	7	7	6
PF 4	10	10	10	9	9
PF 5	10	9	8	6	6

G: Government School *M: Male* *P: Private School* *F: Female*

ANALYSIS

The data are compiled in the following histograms and a comparison can be drawn between the students.

1. Fig. 5.1 compares the score of boys and girls in a all five parameters in government school.
   Maximum score: 50
   Minimum score: 0
   The results indicate that in government school, boys outshined in all five Newman's processes of problem-solving.
2. The histogram given in Fig. 5.2 compares the score of boys and girls in all five parameters in a private school.
   Maximum score: 50
   The results indicate that in private schools, girls outshined boys in all five processes.
3. The histogram given in Fig. 5.3 compares the scores of government and private school students in all five parameters.

- The results indicate that children of private school performed better than those of government school when it came to solving word problems and going step-by-step through Newman's five parameters.
- The maximum gap between private and government school children was in the encoding process.
- Overall, the process of reading was achieved by most of the students (both girls and boys).
- But the maximum gap between girls and boys is in the processes of transformation and process skill.
- There is a significant lag in the process of comprehension in both the schools, which

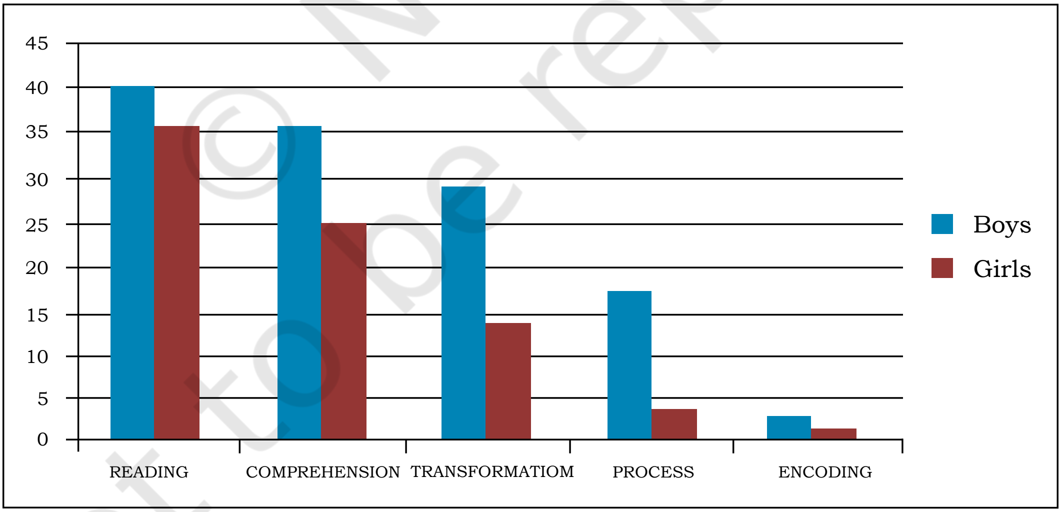


Fig. 5.1: Histogram comparing the scores of boys and girls of government school on Newman's parameters

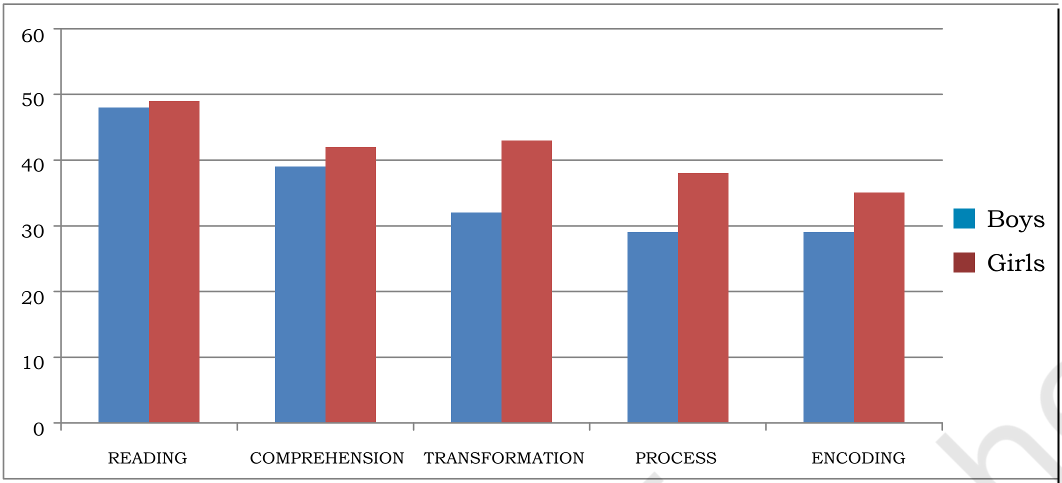


Fig. 5.2: Histogram comparing the scores of boys and girls of private school on Newman's parameters

Maximum score: 100

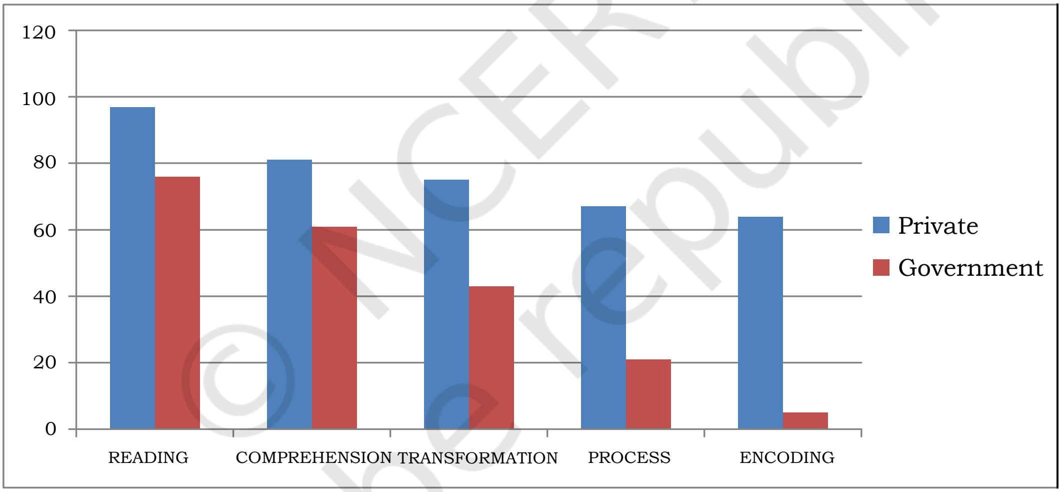


Fig. 5.3: Histogram comparing the scores of private and government school students on Newman's parameters

resulted in lags in other consecutive steps.

- The overall analysis among five parameters is

Reading process (most efficiency)> Comprehension > Transformation> Process Skill > Encoding (least efficiency)

CONCLUSION

Students have the habit of attempting to solve word problems using only one heuristic. They do not show flexibility in seeking to solve the problems using more than one heuristic. The study highlights the processes, which students go through while working on word problems. Isolated questions without context create no interest in students and they are not able to solve the problems. It is seen that students are better in reading and comprehension processes but when it comes to converting or translating the words in mathematical terms to reach a solution, they are not able to do so. This is of great concern.

Students must possess the required knowledge and be able to use appropriate skills to solve problems. As the questions given to the students were in coordination with the National Curriculum Framework – 2005 and NCERT textbooks, there was still a lack of conceptual knowledge in some areas. Algorithmic knowledge, linguistic knowledge, conceptual knowledge, schematic knowledge and strategic knowledge are vital traits of developing problem-solving ability. For mathematics teachers to assist their students in developing problem-solving skills, it is essential that they are aware of their difficulties first. The study suggests a conceptual framework, which can help teachers to facilitate students on working with word problems in a better way. The framework consists of four domains:

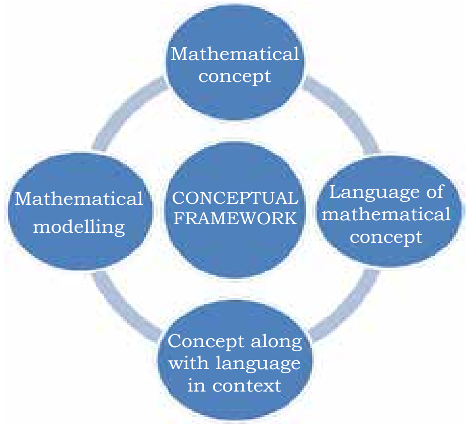


Fig. 5.4: Suggestive conceptual framework

- Mathematical concept includes introduction of concept, clarity in all concepts.
- Language of mathematical concept includes symbolic, verbal, pictorial, everyday context language.
- Concept along with language in context includes concept with real life.
- Mathematical modelling includes decoding, identifying the concepts and mathematical language, developing suitable algorithm, problem solving procedure, translating problem into solution.
- The framework is an attempt to help teachers and curriculum makers to build an insight on the child's perspectives of solving word problems.

REFERENCES

NCERT. 2005. National Curriculum Framework. NCERT. New Delhi.

NCTM. 1990. Teaching and learning mathematics. NCTM.

Polya, George. 2004. *How to Solve it.* Princeton University Press.

6

Peer Learning in Mathematics among Primary School Children

Ritu Giri*

"Learning is the acquisition of knowledge or skill through study, experience or being taught."

Oxford Advanced Learner's Dictionary

Humans and animals both have the ability to learn. Children and adults learn in various social contexts by exchanging ideas and thoughts. An individual does not always construct knowledge by himself/herself, rather this construction of knowledge depends on various social interactions, in which he/she engages with peers or adults. Similar to the importance of social learning, benefits of peer interactions have also been appreciated for long. But still, it can be seen that teachers do not make use of peer learning in classrooms. It may be because they do not consider it practical enough to be used in the learning process. And, especially, if we talk about mathematics, the percentage of peer interactions and teachers' attempt to generate such interactions remains considerably low.

But with the changing time and context, this view is also changing. The way mathematics was perceived is modifying and it is no longer considered to be an absolute or fixed body of knowledge. As per the National Curriculum Framework (NCF)–2005 vision for school mathematics, "Children should enjoy mathematics rather than fear it." This vision of mathematics provides us with an idea of a learning environment, where a child constructs knowledge through active participation and by interacting with others. This view takes us somewhere to the concept of peer or collaborative learning. This study attempts to focus on the importance of peer learning in mathematics classroom. Here, an attempt has been made to see how it works and

* *Primary Teacher*, Sant Nagar School, Varanasi

influences a child's learning—both cognitively and socially.

Theoretical Underpinnings

Peer learning is evolving, developing and being implemented in various ways in different areas. It is beneficial for both an individual a group. There is considerably less work done on peer learning, specifically referring to mathematics classroom.

"…Generally, 'peers' are other people in similar situation to each other who do not have a role in that situation as teachers are expert practitioners. They may have a considerable experience or expertise or they may have relatively little. They share the status as fellow learners and they are accepted as such…" (Boud, 2002).

"…It (peer learning) here suggests a two-way, reciprocal learning activity. Peer learning should be mutually beneficial and involve sharing of knowledge, ideas and experience between the participants. It can be described as a way of moving beyond independent to inter-dependent or mutual learning." (Boud, 1988).

Some other works on peer learning include—cooperative learning and motivation (Abass, 2008); cooperative learning: heterogeneous v/s homogeneous grouping (Sunarti, Das and Rai, 2006); research and rational cooperative learning structures (*Kagan Online Magazine*, Winter 2001); and cooperative learning in mathematics (Lekin and Zaslavsky, 1999). Collaborative learning enhances critical thinking (Anuradha A. Gokhale, 1995), and so on.

In a meta-analysis of 158 studies, Johnson and Johnson report that the current research findings present an evidence that cooperative learning methods are likely to produce positive achievement results. In various cases, the achievement levels are considerably higher when tasks involving peer interactions are used compared to individual ones.

Sample and Methodology

The present study focuses on the importance of peer learning in a mathematics classroom. Being empirical in nature, it is quasi-experimental, which is used to get an idea of the casual impact of various interventions in the form of tasks on the research group.

The students selected in the study were from an MCD school in GTB Nagar, New Delhi. A sample of 46 students belonging to Section B of Class III was selected. Of these, 21 were girls and 25 were boys. Keeping in mind the curiosity of the children to interact and the nature of mathematics in higher grades, which becomes more abstract, a primary section was chosen as sample.

The study took about 48 days. Various peer learning tasks were identified for mathematics class from different sources. The tasks were selected with a motive to generate the need of interaction among students and this could be successfully completed only with their participation. Also, the tasks were based on various topics in mathematics, such as addition, subtraction, shapes, etc. The tasks involved both homogenous and heterogeneous grouping based on the nature of activity. Once the tasks were selected, they were then applied on the selected group. The interactions were carefully and regularly observed, and records of dialogues were made.

The data were then analysed in the light of development of mathematical concepts, mathematical processes, math talk and other things based on analysis interpretations, after which conclusions were drawn.

Analysis and Findings

During analysis, it was seen that in collaborative tasks, peers acted as equal partners, worked in collaboration with each other and not as authority figures. They engaged in sharing of ideas, giving feedback, drawing interpretations and taking joint decisions. In various tasks executed, the learners were provided with opportunities not only to discuss and share their understanding but also engage in math talk through which they not only learnt to communicate but were also able to explain, justify, articulate, defend and reflect their mathematical thinking with confidence. The students, here, had the scope to discuss and share things, therefore, they could be seen participating in classroom activities.

Several instances could be seen where the students not only engaged in quick mental computations but also explained to each other why and how of different things. They were engaged in explaining their reasoning, which gave rise to talks related to math. The students' ability to articulate their responses showed their clarity of concepts. An awareness of each other's reasoning and arriving at a consensus with peers helped the students to reconsider concepts, and solve the problem at hand. The students were happy that their responses were given a place in the discussion and they were motivated to take part in class. In groups, the students were seen adopting various informal strategies while discussing with their peers, which made various conceptual clarifications to the group. Also, a sense of responsibility could be seen as all students were working to help their team.

Working with the group for its benefit acted as a motivating factor for them. Also, verbalising while acting out helped other group mates to know what a child was thinking and enhance their understanding. In a single task, different students came up with different strategies, which broadened their understanding. The students could be seen discussing among themselves and reaching out at an agreement for the benefit of the group. Leaving their individual interests for the benefit of the group was their aim and in this process they were learning to accept each other's opinions. Time-to-time, they exchanged their positions and took leadership roles. The students could be seen planning, organising or dividing the task. Hence, they worked upon their organisational and time management skills. They became happy about their contribution in the task, which helped them to build their self-esteem. The students helped each other to comprehend the problem, interact informally and provide strategies to deal with it using different forms of representation. In various instances, the students brought about their social context in the classroom, which provided various points of view. They analysed the points and used their understanding to adopt them.

They, in this process, practically strengthened various mathematical concepts and enhanced their cooperation skills. It not only helped children with similar cognitive abilities but also indentified the gap in their knowledge. During these interactions, each child contributed differently and something productive. Not everything in the conversation is always worth, sometimes off-task conversation could also be observed. Both boys and girls were engaged in cooperative tasks. It is important on the part of the teacher to listen to and communicate with the students during these tasks.

Conclusion

Peer learning accompanies various advantages and implications. Therefore, it is important for the teacher to be aware of those. Teachers with such a belief are likely to build knowledge communities, improve teacher-pupil talks and help improve pupil-pupil talk. It is not just important for the teacher to provide discussion time but also engage his/her pupils in purposeful talks for productive output. This does not require much effort but just some modifications in instructions and the teacher's own motivation to give something worthwhile to his/her pupils.

REFERENCES

Abass, F. 2008. Cooperative Learning in Motivation. *Aichi University Bulletin*, Nagoya, Japan.

Boud, D. 2001. *Introduction: Making the Move to Peer-learning.* In D.Boud.Cohen, Ruth and Sampson Jane (Ed.). Peer learning in higher education: Learning from and with each other. Kogan Page Ltd. London. pp. 1–17.

———. 2002. What is peer learning and why is it important? *Tomorrow's Professor.* Accessed from http://web.stanford.edu/dept/CTL/Tomprof/posting/418html

Das, Samsudin and Nootan Rai. 2006. *Cooperative Learning: Heterogeneous vs. Homogenous Grouping.* CHII St. Joseph's Convent, Singapore.

Gokhale, Anuradha. 1995. Collaborative learning enhances critical thinking. *Journal of Technology Education.*

National Curriculum Framework. 2005. NCERT, New Delhi.

Zaslavsky and Roja Leikin. 1999. Cooperative Learning in Mathematics. *The Mathematics Teacher.* Vol. 92. No. 3.

7

Understanding Rational Numbers[1]: Are the Textbooks helping?

Sona V. Andrew*

Introduction

Classroom discussions with students of pre-service elementary education programme (B El ED[2]) on the concept of rational numbers has made me reflect on their understanding of rational numbers. During one of the discussions with third-year B El ED students, I asked them what is a rational number? The students gave me the definition of rational number that they had learnt in school. They said, "A rational number is any number in the form a/b, where b≠0", while "an irrational number is a number that cannot be expressed in the form a/b, where b≠0".

The students were then asked, if 1/3 is a rational number? They answered in the affirmative. They were, then, asked where will you place 10/3? Is it a rational or an irrational number? The students answered, "It is a rational number!" Next, they were asked where will you place 3.3333? Is it a rational or an irrational number? The students answered: "It is an irrational number". Then, the following was deduced from the above interaction

If 10/3 = rational number, and 3.3333... = irrational number

Then, 10/3 = 3.3333... = irrational number, and 10/3 = rational number.

Therefore, all rational numbers = all irrational numbers.

That is when the students realised that there was something wrong in the

1. Students in elementary school may be more familiar with the term 'fractions' as compared to 'rational numbers'. However, in this article, the term rational number is used as it captivates the meaning of the term fraction and many more concepts in it. The rationale of this will become clearer to readers when the theoretical framework is discussed.
2. B EL ED is a four-year integrated teacher education programme, which was started by the University of Delhi in 1994. The course continues to prepare reflective pedagogues in elementary school.

* *Associate Professor*, Department of Elementary Education, Jesus and Mary College, University of Delhi

deductions that were made. I reflected as to why the students were not able to make connections? Such experiences were not specific to a particular batch of students. This understanding of rational numbers is reflected across batches. The questions that came to my mind were — why have not these students made a connection between the two representations of rational number? If the students are unable to recognise the same number in the representations, then have they understood the concept of rational number? Why is it that even after engaging with the concept of rational numbers since elementary school, the students are not able to move beyond the definition of rational number?

The answers to the questions raised lies in the experiences that the students had gained in school while engaging with rational numbers. The classroom experiences of the students could be in tandem with the experiences explicated in textbooks. Just studying the textbooks may not present a complete picture of classroom engagements of the students. However, they may help develop an understanding of the content that is likely to be discussed in the classroom.

Derek Stolp (2010), while discussing the dependence of a teacher on textbooks, shares a teacher's comments on textbook: "The textbook provides a map and if we (teachers) follow it, we won't get lost. The more faithfully we follow this map, we believe the better mathematicians we will become."

This reflects the teacher's dependence on textbooks, and thus, it is imperative to understand the content of the textbooks to see whether they are helping the students acquire an understanding of rational numbers beyond the definitions and procedures. The textbooks will be analysed to comprehend how they unfold the concept of rational numbers. As per the National Curriculum Framework–2005, the students are to engage with rational numbers[3] from Class IV to VIII. This study chooses to focus on the initial years of the students' learning of rational numbers. Thus, NCERT mathematics textbooks of Class IV and V were studied to understand the experiences being provided to the students.

What are Fractions and Rational Numbers?

Fraction is a term that is used in primary classes to address part-whole relations of rational numbers. Peter Gould (2005) observes that the greatest advantage of fraction or part-whole models is that it is a readily available option for teachers and the text to introduce the symbolic notation to students. Thus, fractions form an integral part of primary school mathematics.

3. The students, as per the National Curriculum Framework (NCF)–2005, are introduced to fractions from Class IV onwards.

The mathematics textbook of Class VII published by the NCERT (2007) defines 'rational number' as a number that can be expressed in the form p/q, where p and q are integers and q≠0. This basic definition of rational number that any number in the form p/q, where q≠0 allows to classify mathematically the concepts of fraction, decimal, measure, ratio and proportion, and percentage under rational numbers, while maintaining the defining characteristics of each of the concepts. The theoretical framework for this collation of concepts under rational number was provided by Thomas Kieren. Kieren (1978) in his research on rational numbers established that the concept of rational number is not a single construct, and in 1980, he established the five ideas of rational numbers, which he called sub-constructs that could form the basis for understanding rational numbers. They were—fractions or part-whole sub-construct, quotient sub-construct, measure sub-construct, ratio sub-construct and operator sub-construct. The details of the theory is explicated under the heading 'The Sub-construct Theory of Rational Number'.

Rational numbers include all natural numbers and integers as depicted in *Fig. 7.1*. Real numbers are larger sets, which contain all rational numbers and numbers, which cannot be represented in the form of p/q, q≠0 (irrational numbers).

The students are introduced to the concept of fractions or part-whole

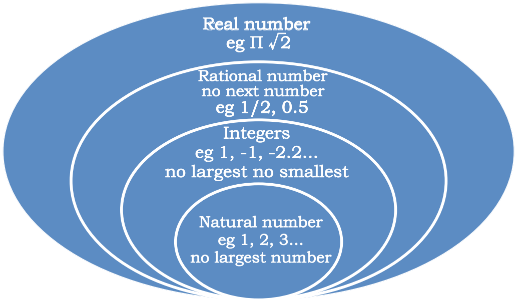


Fig. 7.1: A simplified model of the extension of number domains from natural to real numbers and changes in the level of abstraction with each enlargement (Merenluoto, et al., 2007)

sub-construct of rational numbers in Class IV (NCF, 2005). However, the term rational number is not introduced to them. The term rational number explicitly features in the curriculum only in Class VII (NCF–2005). The textbook beyond elementary school does not have rational numbers as an explicit chapter. It is assumed that students beyond elementary school are proficient in this concept. Through the elementary years (Classes IV to VIII), the textbooks expose students to halves and quarters, parts and whole, fraction, ratio and proportion, decimal, and comparing quantities.

The Sub-construct Theory of Rational Number

According to the framework by Kieren (1976), the Sub-construct Theory of Rational Number looks at rational number as a synthesis of five concepts. This is the uniqueness of this theory as it allows a comprehensive perspective of rational number. Thus, to understand the concept of rational numbers, the students must engage with each sub-construct. The focus at this point should be to comprehend what each sub-construct signifies.

Fraction or part-whole sub-construct: Part-whole sub-construct assigns a number of equal parts of a unit (unit can be a continuous quantity or a set of discrete objects) out of the total number of equal parts into which it is divided.

Example: Identifying 3/4 in a whole, which has been partitioned into eights (Charalambous, et al., 2007)

Quotient sub-construct: Partitioning or dividing into equal parts is the basis for a rational number to be understood as a quotient. In particular, the rational number x/y indicates the numerical value obtained when 'x' is divided by 'y', where 'x' and 'y' are whole numbers (S. Lamon, 1999).

Example: Three pizzas are evenly divided among four children. How many pizza pieces will each child get? (Behr, et al., 1993)

Measure sub-construct: Measurement interpretation is pointed towards the use of number line as a physical model. The reason for the term 'measurement interpretation' is that rational numbers are defined as measures — the unit of measure is the distance on the number line from 0 to 1 (Kieren, 1976).

Example: In a given line segment from 0 to 1, identify where 1/3 would lie.

Ratio sub-construct: The ratio interpretation of rational numbers supports the notion of comparison between two quantities. Thus, it is considered comparative index rather than a number (Charalombos, 2007).

Example: Three boys share one pizza and seven girls share three. Who gets more pizza pieces, a girl or a boy? (Lamon, 1999)

Operator sub-construct: In the operator interpretation of rational numbers, rational numbers are regarded as functions applied to some number, object, or set. For example, 3/4 is thought of as a function applied to some number. The significant relationship is the comparison between the quantities that is acted upon. The operator defines the relationship quantity out or quantity in (Lamon, 1999).

Example: Sahil has 3/4 as many toy cars as I have. I have 40 toy cars. How many does Sahil have?

Challenges to the Learning of Rational Numbers

Moss and Case (1999) argue that the domain of rational numbers is considered to be one of the most complex mathematical concepts in elementary school because understanding rational numbers requires a conceptual shift, i.e., numbers must be understood in multiplicative relations. Since the concept of rational numbers is embedded in the multiplicative conceptual field, it has a distinct character as compared to the whole number concept and its operations (Vergnaud, 1983). Hence, many of the successful strategies, which the students may have developed for whole numbers, will not work for rational numbers, like the largest number is the longest number.

Another example, is '2 is less than 3' however '1/2 will not be less than 1/3' on the contrary '1/2 is more than 1/3'. Researchers (Moss, 2005; Ni and Zhou, 2005) have identified this to be one of the major causes of difficulty for students. They named such errors arising out of applying natural number knowledge in situations when it is not appropriate as natural number knowledge interference. Another challenge for students is that they are taught while learning the properties of natural numbers that there is no natural number between any two consecutive numbers. But in case of rational numbers, this property is not true. On the contrary, there are infinite number of rational numbers between any two consecutive natural numbers (Lamon, 2012; Stafylidou and Vosniadou, 2010).

The students face challenge in the representation of rational numbers because of multiplicity in the representation of a rational number — diagrammatically and symbolically. When rational numbers are to be represented diagrammatically, there are three options — set model, area model and linear model. According to Jeremy Kilpatrick, et al. (2001), the task for students while trying to understand rational numbers is to recognise these distinctions, and at the same time, construct relations among them to generate a coherent concept of rational numbers. And, when a rational number is represented symbolically, there are many options, like p/q notation, decimal representation, percentages, and equivalent fractions, for example, the number 1/2 can be represented as 0.5, 50 per cent, 0.50, 0.500, 2/4, 3/6, 4/8, etc. The list is endless. This multiplicity in representation, symbolically and diagrammatically, offers many challenges to the students and all are not able to overcome it.

The question that needs to be answered is — are the students given enough opportunities to engage with the concept of rational numbers to develop a comprehensive understanding of the concept. The textbook must provide space and opportunities to students to challenge the misconceptions they develop from 'natural number knowledge interference' and multiplicity in the representation of the same rational number.

Textbooks and Rational Number Sub-constructs

When textbooks were scrutinised vis-à-vis the sub-construct theory of rational numbers, it was observed that only three of the five sub-constructs were represented namely — fraction or part-whole sub-construct, measure sub-construct and operator sub-construct. Ratio and quotient sub-construct were missing from primary class textbooks. However, unlike other sub-constructs, ratio sub-construct has separate chapters across textbooks for middle school. Among the sub-constructs represented in the textbooks of Classes IV and V, it was observed that fractions or part-whole sub-construct was given the most exposure.

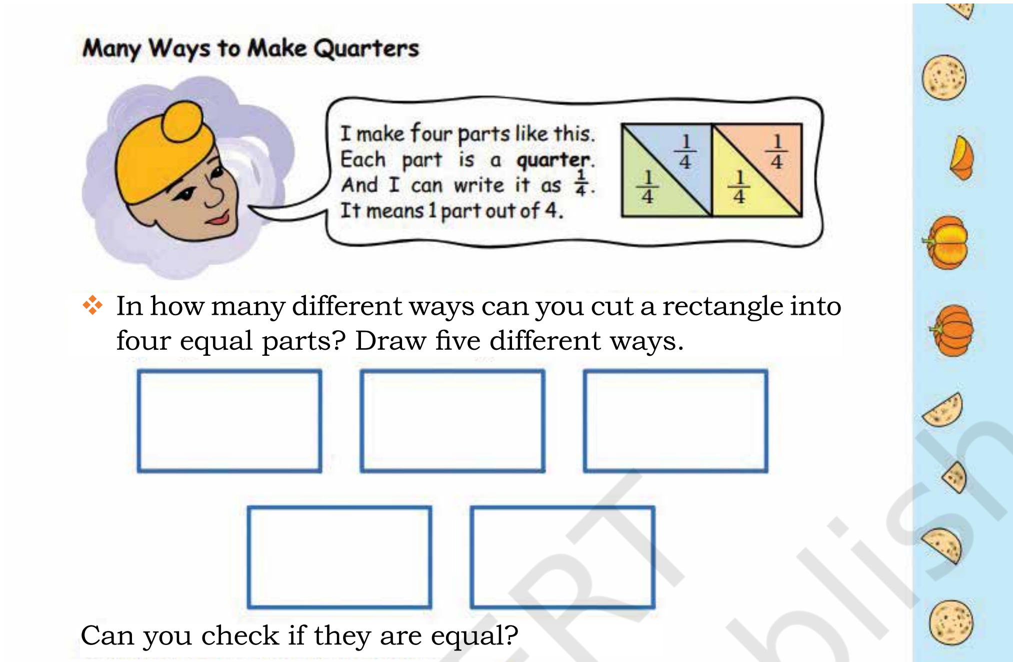
Researchers (Seibert and Gaskin, 2006; and Van de Walle, 2013) inform that the actions of partitioning (3/4 is three one-fourths, where 1/4 is the result of dividing the whole into four parts, and 1/4 is one of the four parts), iterating (3/4 is three one-fourths, where 1/4 is the amount such that four of the one-forth join together to make a whole), and sharing (sharing of four apples between two persons) have been recognised as important to the understanding of part-whole relations.

The textbooks have several examples of partitioning task (a situation where a whole is divided into parts) but those of sharing task (a situation where a whole or more is shared among two or more people) are few (one example of sharing *halwa* is on page 63, Math Magic, Textbook in Mathematics for Class V, 2008, NCERT). Iteration task (counting using a part of a fraction to arrive at a whole) is represented in the set model in both the textbooks by using currency, and weights and measures as contexts. Also, there is one question in the area model (page 63, Math Magic, Textbook in Mathematics for Class V, 2008, NCERT), wherein the students have to find the 'whole from a part'. Instances of completing the picture are also introduced in the textbook of Class IV. But are the students able to connect the act of completing the picture to iteration?

Fig. 7.2: An example of iteration by completing the figure

An important aspect of the textbook is that it encourages students to think that there are multiple ways to arrive at a solution, like in the case of dividing a figure into four parts. The students are asked to divide the figure in multiple ways. They are explicitly asked to draw lines along which the figure can be divided. Space is also provided for the students to explore several options. Thus, the students are made to think of multiple ways of dividing a figure, thereby, forcing them to explore different ways to divide the same.

There are examples of all three models of the diagrammatic representation — set model (all tasks involve countable number of objects), area model (all tasks involve sharing something that can be cut into pieces), and linear model (all tasks involve number lines, these are therefore,


Many Ways to Make Quarters

I make four parts like this. Each part is a **quarter**. And I can write it as $\frac{1}{4}$. It means 1 part out of 4.

$\frac{1}{4}$ $\frac{1}{4}$ $\frac{1}{4}$ $\frac{1}{4}$

❖ In how many different ways can you cut a rectangle into four equal parts? Draw five different ways.

Can you check if they are equal?

Fig. 7.3: An example from Class IV

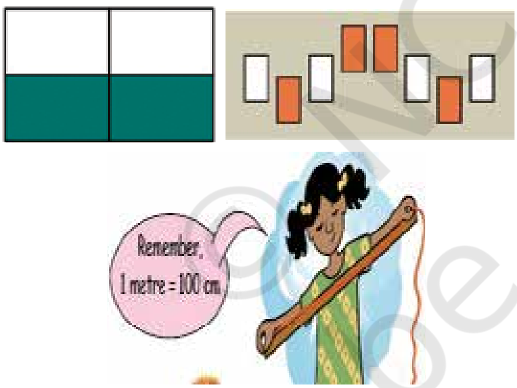


Fig. 7.4: Examples of the three models of representation

tasks of measure sub-construct) — in the textbooks.

Given above are examples on area model (page 101, Math Magic, Textbook in Mathematics for Class IV, 2007, NCERT), set model (page 102), and linear model (page 103). The textbook, however, tends to focus more on the area model, and there is relatively less space for tasks of set model. There is a little space for tasks in the linear model or measure sub-construct. There is only one instance in the Mathematics textbook of Class IV of a child using a measuring tape, which qualifies to be a task on measure sub-construct. Thus, the only engagement that the students have with measure sub-construct is in the textbook of Class IV, and that too, just one activity.

Operator sub-construct does have a presence in the textbooks. There is a story of Birbal (page 55) and a task on Ramu's vegetable field (page 58) in the Mathematics textbook of Class V. Through these examples, the students get an idea of operator sub-construct and these will help them understand the concept of multiplication of rational numbers.

Like ratio sub-construct, there was no instance of quotient sub-construct in these textbooks. However, for ratios, there is an entire chapter in the textbook of Class VI. But for quotient sub-construct, there is only a small section on the division of fraction, which has been addressed mechanically in the textbook for Class IV. Examples like '4/5 can also mean the share of one person when five persons are sharing four *poories*', can be given to students. Such tasks can be introduced diagrammatically in the primary classes itself. Should the students wait that long to get introduced to examples like these for quotient sub-construct? If examples like these are introduced in earlier classes by encouraging the children to represent the portions diagrammatically, while dealing with small numbers, they are likely to learn about the division of fractions much earlier than in Class VII. An option available for the textbook is to include sharing tasks under fraction sub-construct and link them to the ideas of quotient sub-construct.

Suggestions

A textbook must provide students with information about all five sub-constructs. Jeremy Kilpatrick, et al. (2001) in the book, *Adding it up,* remark that students will be able to generate a coherent understanding of the concept only if they are able to "recognise these (five sub-constructs) distinctions and at the same time, construct relations among them". The textbook (Class V, NCERT) has not addressed all sub-constructs of the rational number sub-construct theory. And in places where the sub-constructs were included, the tasks were few.

Thomas E. Hodges, et al. (2008) in their article titled 'Fraction Representation: The not-so-common Denominator among Textbook' comment that since the textbook lacks multiple representations, set model, area model and linear model to support students' learning, "therefore, it is up to the teachers to encourage students to use and reflect on their use of representations" (page 82). However, the textbook should have examples of all types of representation rather than putting pressure on teachers to find tasks to complete the work done in the textbook on all aspects of rational number. Van de Walle (2013) in the book titled *Teaching student-centered Mathematics* (page 295) observes: "As a teacher, you will not know that whether they really understand the meaning of a fraction, such as 1/4 unless you have seen a student represent one-fourth using area, length and set models."

A textbook must have space for students to engage with issues that emerge from natural number knowledge interference. Opportunities must be provided to them to challenge their understanding of the concept of rational numbers with the understanding they have of natural numbers. If these misconceptions are not challenged in the early years of engaging with rational numbers, they may get crystallised rather than the idea of rational numbers.

Conclusion

The above analysis reveals that the textbooks (Class IV and V, NCERT) is far from providing a holistic understanding of rational numbers. The ratio and the quotient sub-constructs are not represented in NCERT textbooks. The textbook must incorporate examples and tasks for ratio sub-construct, quotient sub-construct, and have more tasks for measure sub-construct. Even though ratio sub-construct and measure sub-construct are addressed in the middle school, the students must be to introduced to these concepts in the primary classes itself.

The ideas that have been described above are an attempt to help teachers understand the concept of rational numbers holistically. The teaching of rational numbers must be in tandem with the sub-construct theory of rational numbers and the textbook must support teachers in this endeavour by including experiences needed to understand rational numbers in the textbook. Tasks of varied nature are necessary to provide an opportunity to students to explore and understand each sub-construct of rational numbers.

References

Charalambous, C.Y. and D. Pitta-Pantazi. 2007. Drawing on a theoretical model to study students' understandings of fractions. *Educational Studies in Mathematics*. Vol. 64. No. 3. pp. 298–316.

Hodges, T.E., J. Cady and L. Collins. 2008. Fraction Representation: The not-so-common Denominator among Textbooks. *Mathematics Teaching in the Middle School*. Vol. 14. No. 2. pp. 78–84.

Kieren, T.E. 1976. On the Mathematical, Cognitive and Instructional. In *Number and Measurement*. Papers from a research workshop. Vol. 7418491. p. 101.

——. 1980. The Rational Number Construct: Its Elements and Mechanisms. *Recent Research on Number Learning*. pp. 125–149.

——. 1993. Rational and Fractional Numbers: From Quotient Fields to Recursive Understanding. *Rational Numbers: An Integration of Research*. pp. 49–84.

Kilpatrick J.J. Swafford, B. Findell, 2001. *Adding it up: Helping Children Learn Mathematics*.

Lamon, S.J. 1999. *Teaching Fractions and Ratios for Understanding: Essential Knowledge and Instructional Strategies for Teachers*. Lawrence Erlbaum Associates: Mahwah, New Jersey.

——. 2012. *Teaching Fractions and Ratios for Understanding: Essential Content Knowledge and Instructional Strategies for Teachers*. Routledge.

Moss, J. 2005. Pipes, tubes, and beakers: New approaches to teaching the rational number system. *How Students Learn: History, Math and Science in the Classroom*, pp. 309–349.

Ni, Y. and Y.D. Zhou. 2005. Teaching and learning fraction and rational numbers: The origins and implications of whole number bias. *Educational Psychologist*. Vol. 40. No. 1. pp. 27–52.

Siebert, D. and N. Gaskin. 2006. Creating, Naming, and Justifying Fractions. *Teaching Children Mathematics*. Vol. 12. No. 8. 394–400.

Stolp, D. 2005. *Mathematics Miseducation: The case against a tired tradition*. R&L Education.

Vamvakoussi, X. and S. Vosniadou. 2010. How many decimals are there between two fractions? Aspects of secondary school students' understanding of rational numbers and their notation. *Cognition and Instruction*. Vol. 28. No. 2. pp.181–209.

Van de Walle, J.A., K.S. Karp, L.A.H. Lovin and J.M. Bay-Williams. 2013. *Teaching Student-centered Mathematics: Developmentally Appropriate Instruction for Grades 3–5*. Pearson Higher Ed., New York. Vol. 2.

Vergnaud, G. 1983. Multiplicative structures. *Acquisition of Mathematics Concepts and Processes*. pp. 127–174.

8

A Study on Anxiety in Upper Primary School Students in learning Mathematics

Kanamarlapudi Venkateswarlu*

Abstract

This study was conducted to find out anxiety in learning mathematics by upper primary school students of Prakasam district in Andhra Pradesh. The investigators adopted normative survey method for the study. A sample of 200 students from upper primary schools was selected by Systematic Random Sampling Technique. Anxiety tool was used for the study. For the analysis of data, descriptive statistics, like mean, standard deviation and 't' test were employed. The findings revealed that a significant difference was found in anxiety in learning mathematics.

Introduction

In this technological era, education is considered as the first step for every human activity. It plays an essential role in the development of human capital and is linked with an individual's well-being, and provides him/her with opportunities for a better living. Academic achievement is considered essential in this age of rapid change in competitions. There are certain factors, which affect students' anxiety in learning mathematics.

Anxiety

An uncomfortable feeling of nervousness or worry about something that is happening or might happen in future.

Statement of the Problem

A study on anxiety faced by Class VII students in learning mathematics

Need of the Study

Without numerical and mathematical evidence, we cannot decide numerous issues in our day-to-day life. Mathematics is the study of

* *Associate Professor,* Department of Post-graduate Education Research Centre, RVRR College of Education, Guntur, Andhra Pradesh

abstractions and their relationships, in which the only technique of reasoning that may be used to confirm any relationship between one abstraction and another is 'deductive reasoning'.

The place of mathematics in modern education must be determined by an analysis of the culture of civilisation of the modern society. The Kothari Commission report (1964–66) points out that the study of mathematics plays a prominent role in modern education. It says, "One of the outstanding characteristics of scientific culture is qualification." Mathematics, therefore, assumes a prominent position in modern education. Mathematics education in schools is emphasised more as it improves concept development, fosters higher cognitive abilities and skills.

Arithemetic includes the study of whole numbers, fractions, decimals, and operations of addition, subtraction, multiplication and division.

Arithmetic ability is used in everyday situations of our life. Hence, it is important in learning mathematics. When arithmetic ability is there, students' achievement will be good. When achievement in mathematics is good, they can go for higher studies easily.

Scope of the Study

The present study focuses on anxiety faced by Class VII students in learning mathematics. A sample of 200 Class VII students was selected. Variables, such as gender, locality, management and medium of instruction were taken into account to conduct the study in schools in Prakasam district of Andhra Pradesh.

Objectives of the Study

The objectives of the study are as follows:

(1) find out the anxiety faced by upper primary school students in learning mathematics
(2) find out the anxiety in learning mathematics by upper primary boy and girl students
(3) find out the anxiety faced by rural and urban upper primary school students in learning the subject
(4) find out the anxiety faced by government and private upper primary school students in learning mathematics
(5) find out the anxiety in learning mathematics by Telugu and English-medium upper primary school students

Review of Related Litarature

Researchers found there were many factors that influenced poor learning in mathematics. Rastogi found that one of the important causes of backwardness in mathematics was the poor command over basic arithmetic skills. Attitudes were closely linked with achievement. When command over basic arithmetic skills improved, attitude towards mathematics became more favourable and achievement in the subject increased. It was also found that there was no significant sex difference in either attitude towards mathematics or achievement in the subject.

M. Chitkaram found that boys and girls of superior ability did not show significant difference between mean score on achievement in mathematics. Girls of average ability scored significantly higher in mathematics than boys.

Sunil Kumar found that the mean scores of urban students have been better than that of rural students. It was also found that OBC students got better achievement scores than SC/ST students.

P. Nirmala found that many factors influenced the academic achievement of students in mathematics at the higher secondary school level.

In the present study, it is observed that mathematics information processing skill, decision making skill and attitude towards the subject have made a significant contribution towards academic achievement in it.

Method of Research

The study involves normative survey method. The investigator selected this method because it is only status study.

Variables of the Study

The variables considered for the study, are as follows:

(1) boys versus girls in upper primary schools

(2) rural versus urban upper primary school students

(3) private versus government upper primary school students

(4) English versus Telugu-medium upper primary school students

Hypotheses of the Study

Upper primary school students do not possess high anxiety in learning mathematics.

Hypothesis 1a: There is no significant difference in anxiety in learning mathematics between boys and girls of upper primary schools.

Hypothesis 1b: There is no significant difference in anxiety in learning mathematics between rural and urban upper primary school students.

Hypothesis 1c: There is no significant difference in anxiety in learning mathematics between government and private upper primary school students.

Hypothesis 1d: There is no significant difference in anxiety in learning mathematics between Telugu and English-medium upper primary school students.

Sample of the Study

After a detailed study of all sampling methods and considering the variables selected for the research work, stratified sampling method was found suitable. In stratified sampling, the researcher has a greater control over sample selection and use.

Two hundred students of Class VII of upper primary school were selected for the study.

Tools of the Study

The selection of suitable tools for conducting the study is important. For conducting a research in any field, we need good research tools for the measurement of the aspects to be studied. Self-made anxiety tool has been used for the study.

Analysis of the Data

The data collected through the use of tools need to be organised, edited, classified and tabulated before analysis and interpretation to get generalisations and draw conclusions.

The aspects of anxiety will be analysed individually. The hypotheses framed for the study will be tested statistically and accepted or rejected accordingly. Statistical procedures, like mean, Standard Deviation (S.D.), critical ratio, chi-square test will be used to analyse the data.

Discussions and Conclusions

(1) The upper primary school students face anxiety in learning mathematics (Table 1.1).

The education system is giving more importance to enrich children's knowledge but is ignoring anxiety among students. Even parents lay more importance to educate their children but ignore anxiety among them. Over-ambition is forced by parents on students, which causes anxiety. This is not a correct situation in the education system. Parents and teachers must work to reduce anxiety level in students. It is only then that the students will achieve good results and positions.

Table 1.1

Sample	*Sample size*	*Mean*	*S.D.*
Whole	200	38.5	6.5

(2) There is no significant difference in anxiety in learning mathematics in boy and girl students of upper primary school (Table 1.2).

The study revealed that there is a slight difference among boy and girl students as regards to anxiety in learning mathematics. Girls have high level of anxiety in learning mathematics than boys. As girls face

Table 1.2

Variable	*Sample size*	*Mean*	*S.D.*	*Difference between means*	*S.E.D*	*C.R.*
Upper primary school boys	100	42.96	8.18	1.2	1.69	0.70*
Upper primary school girls	100	44.16	8.71			

* Not significant at 0.05 level

physical and psychological problems, it leads to development of anxiety levels, thereby, making mathematics learning difficult.

(3) There is no significant difference in anxiety in learning mathematics in rural and urban upper primary school students (Table 1.3).

Table 1.3

Variable	*Sample size*	*Mean*	*S.D.*	*Difference between Means*	*S.E.D*	*C.R.*
Rural upper primary school students	100	49.42	4.78	0.18	1.04	0.17*
Urban upper primary school students	100	49.69	5.61			

#Not Significant at 0.05 level

Both rural and urban school students possess the same level of anxiety in learning mathematics as they follow the same instructions. Generally, rural students face high anxiety in learning mathematics but urban students also face the same problem as they are not able to concentrate in the subject.

(4) There is a significant difference in the anxiety level in learning mathematics between government and private upper primary school students (Table 1.4).

The study revealed that government upper primary school students possesed higher level of anxiety than those studying in private school. Special classes mould a child in every aspect, paving the way for character building. The classes are helpful to students as they keep the children away from stress, strain, fear, etc. If the same training is provided to government school students, it will be helpful to them and build their confidence, leading to successful mathematics.

(5) There is a significant difference in anxiety in learning mathematics in English and Telugu-medium upper primary school students (Table 1.5).

Table 1.4

Variable	*Sample size*	*Mean*	*S.D.*	*Difference between means*	*S.E.D*	*C.R.*
Government upper primary school students	100	49.42	5.28	3.88	0.99	5.93
Rural upper primary school students	100	43.54	8.42			

When maths is taught in the regional language, Telugu, the students can understand the theme better compared to English-medium secondary school students. The medium of instruction is a major problem for upper primary school students in learning mathematics.

Suggestions for Further Research

(1) Studies may be taken up to identify the other factors affecting interest in mathematics.

(2) Studies may be conducted to find out the relationship between the anxiety of various management schools.

Table 1.5

Variable	*Sample size*	*Mean*	*S.D.*	*Difference between means*	*S.E.D*	*C.R.*
English-medium upper primary school students	100	48.36	5.84	7.8	1.32	2.12*
Telugu-medium upper primary school students	100	45.56	7.31			

* Significant at 0.05 level

REFERENCES

Dandekar, W.N. 1968. A Textbook of Educational Psychology. *Indian Educational Review.* Vol. 3. No. 2. pp. 217–220.

Desai, S.D. 1979. A study of classroom ethos, pupil motivation and academic achievement (Ph.D. thesis). M.S. University of Baroda, India.

Garrett, H.E. 1973. *Statistics in Psychology and Education.* Vakils Feffer and Simon Pvt. Ltd. Bombay.

Guilford, J.P. and B.G. Fruchter. 1987. *Fundamental Statistics in Psychology and Education* (6th Ed.). McGraw Hill, New York

Das, J.P. 1985. *Textbook of Psychology.* Arnold Heinemann Publications Ltd., New Delhi.

John W. Best and James V. Khan. 2004. *Research in Education.* Prentice-Hall of India Private Limited, New Delhi.

Karim and Sahidur. 2015. Factors contributing dropout among girls. *European Journal of Research and Reflection in Educational Sciences.* Vol. 3. pp. 25–36.

Koul, L. 1993. *Methodology of Educational Research*. Sterling Publishing Pvt. Ltd, New Delhi.

Greeta, A. 2006. Learning Abilities and Achievement in Mathematics. *Edutracs*. Neel Kamal Publications, Hyderabad.

Mangal, S.K. 1993. *Teaching Mathematics*. Prakash Brothers, Ludhiana.

Packien, S. 1983. *Teaching Modern Mathematics*. Doaba House, New Delhi.

9

Ramanujan through a Biopic

Varada Nikalje*

Say the words 'mathematical genius', and the image that comes to the mind is that of a person scribbling apparently meaningless but complex mathematical calculations all over a blackboard. Such images reinforce the idea that mathematics involves long calculations, and that it is too complex for ordinary mortals, and that it is an abstract discipline not connected with practical life — all of which are untrue. As a well-known mathematics professor puts it, "The hope of improvement in mathematics teaching, whether in schools or in colleges, lies mainly in the possibility of humanising it. It is worthwhile to remember that our pupils are human beings...to humanise the teaching of mathematics means to present the subject, so as to interpret its ideas and doctrines, that they should appeal, not merely to the logical faculty, but to the power of interest of the human mind". (Keynes, 1912).

One laudable effort to humanise mathematics and mathematicians is the film, *The Man who knew Infinity*. A British biographical film released in 2015, it traces the life of Indian mathematical genius Srinivasa Ramanujan from childhood to his stay in England, and his death at the age of 32.

Ramanujan was born on 22 December 1887. He was often on the verge of starvation. He worked in lowly jobs, where fortunately, his employers noticed his skill in mathematics. He was then given basic account work. Ramnujan interacted with two college students (who had rented a room in his house) and absorbed all mathematical concepts they knew. Gradually, he began to write on mathematics, and his impressed employers, sent him with letters of introduction to various professors in the field. A meeting with V. Ramaswamy, founder of the Indian Mathematical Society, proved to be the turning point in his life.

* *Associate Professor*, Department of Elementary Education, NCERT

Correspondence with Professor G.H. Hardy at the Cambridge University followed, and the rest is history.

These details of Ramanujan's life indirectly throw light on how the Indian society viewed him, and the society's attitude to geniuses, in general. The film focuses on how the prodigy struggled even as a child for his daily bread, how he was ridiculed by others, how his immersion in mathematics made him fail in other subjects, and how frequently he fell ill due to poor nutrition.

Cultural differences are dealt with great sensitivity in the film, contributing to the larger concept of humanity. Ramanujan was in England at the time of World War – I, when food was scarce and essential items were rationed. His traditional upbringing as a strict vegetarian compounded his problems. The harsh winters made him homesick. What added to the feeling of alienation was that he received no letter from his wife. We later come to know that his mother had hidden all the letters that he had sent from England in a drawer — away from his wife's sight.

The relationship between Ramanujan and his mentor Professor G.H. Hardy is beautifully portrayed. Despite their differences in nationality, religion, culture, temperament and even their approach to mathematics, a deep and lasting bond was formed. Hardy was an atheist, while Ramanujan came from a religious background. Hardy had crossed milestones in his career, Ramanujan was a self-taught mathematical wizard. Hardy believed in mathematical rigour and the indisputable nature of proof, Ramanujan had blinding flashes of insight, in which he could 'see' proofs of theorems. The ultimate insight of course is Hardy's, in discussing the diamond in the rough.

There are memorable moments in the film that tug one's heart. When Ramanujan sees the snowfall for the first time in his life, looking upward to allow snowflakes to drift on to his face, when his wife finds his letters and realises the machinations of her mother-in-law, when Ramanujan falls ill and begins to cough blood — all these provide an emotional contrast to the rational mind whose fascinating theories inspired a number of researches decades later. It is heartbreaking to see that due to non-access to knowledge of achievements in the field of mathematics, Ramanuajn spent months re-discovering already established theorems.

It is no wonder that the film took 10 years to be made. As with all biographical films (biopics), the challenge was that of authentic portrayal. Since a biopic attempts to tell a comprehensive narration of a person's life, or provide a concise view of the historically important years of his/her life, it should show actual people whose actions and views are documented and in public domain. *The Man who knew Infinity* succeeds in portraying a prodigy without undue sentimentality, with a gentle understanding of Ramanujan's oscillation between the struggle for existence and his love for mathematics.

More important is the attitude towards mathematics through which viewers retain that it can be learnt by anyone and is accessible even to a person from a 'non-maths' field. This is a difficult impression to convey, particularly because people, in general, tend to think of mathematics in relation to practical life only. The National Curriculum Framework–2005 states, "The narrow aim of school mathematics is to develop useful capabilities, particularly those relating to numeracy — numbers, number operations, measurements, decimals and percentages." Yet, the realm of mathematics is far beyond this. Ultimately, the aim is "to think and reason mathematically, to pursue assumptions to their logical conclusion and to handle abstraction". Ramanujan's mind devoted itself solely to mathematics, making major contributions to mathematical analysis, number theory and infinity. Ethnic prejudice, housing problems, even failing health did not matter to him.

The film brings out the almost unbelievable achievement of a poor Indian boy walking up the steps of Cambridge University, where only a privileged few were allowed in the nineteenth century. On the 125th birth anniversary of Ramanujan, the Indian government declared that 22 December would be celebrated as the National Mathematics Day every year. This would certainly focus public attention on the discipline and on the mathematical genius. Nevertheless, the fact remains that there are still a number of children who for no fault of theirs are denied access to education. A matter of concern is the persistence of perceptions that create conditions of non-access to education, like girls 'cannot' learn science; 'inferior' castes should not be given education; differently abled children need 'special' education, and so on.

One cannot help but wonder how many Ramanujans are amongst them.

10

Mathematics Laboratory: A Link between Concrete and Abstract

A.K. Wazalwar*

Mathematics occupies an important place in the school curriculum. The study of mathematics not only helps in disciplining the mind but also acts as a catalyst in developing the power of thinking and reasoning, which is the prime goal of every individual. It plays an even greater role in technological and scientific advancements of the present day techno-savvy society.

Despite its acute necessity, the largely deductive and abstract nature of mathematics makes it appear a dull and difficult subject. As a result, most students lose interest in the subject and avoid it. It is necessary to remove this fear for mathematics from the minds of students. They need to be motivated. Their interest in mathematics needs to be aroused and nurtured. This needs to be done right from the elementary stage of learning.

Abstract mathematical concepts evolve from concrete concepts and come through personal experiences. The structure of modern mathematical theories rest on the basic and elementary concepts derived from personal experiences with concrete objects. While analysing a mathematical problem, it can be seen that its comprehension and ultimate solution hinges around the correct perception of objects in the physical situation involved in the problem.

Therefore, the journey towards abstract ideas in mathematics can begin only when something concrete is put in the hands of students. This will equip them with the basic ideas of the concepts in a better way. Thus, mathematics should be learnt by doing rather than simply by reading. This 'doing' of mathematics requires a suitable place for performing mathematical activities. A well-equipped mathematics laboratory is a suitable place for the same.

A laboratory can instantly motivate students and create an environment for mathematics learning.

* *Professor*, Department of Education Kits, NCERT

It can foster mathematical awareness, skill-building and learning by experiences in different branches of mathematics, such as algebra, geometry, mensuration, trigonometry, calculus, coordinate geometry, mechanics, etc. This is the place where students can learn certain concepts using concrete objects.

They can verify many mathematical facts and properties using models, measurements and other activities. A mathematics laboratory enables a student to verify or discover some geometric properties, using models, measurements, paper cutting, paper folding, etc. It can help the student use different tables and ready reckoners in solving some problems. The students can listen to or view certain audio-video cassettes and CDs related to different mathematical concepts. For example, the importance of different congruence criteria of triangles can be made known to the students of upper primary classes by using different modes of handling concrete objects. One student can be asked to draw a triangle. Then, he/she tells other students about the elements of the triangle drawn. The other students should draw triangles, which are exactly the same as this triangle (i.e., a congruent triangle). The original triangle will not be shown to the other students. Suppose, he/she first tells them only the length of one side of the triangle. Accordingly, the other students will try to draw triangles only with the length of the side told to them. The triangles drawn by the other students may not match with the original one. The first student, then, tells them about the length of other side of the triangle. The game continues till all students are able to draw the triangle similar to the one drawn by the first student. At some point of time, the students realise that either three or two sides and the included angle between them, and so on, need to be given to them to get an exact copy of the original triangle. The students understand the importance of the SSS (side-side-side), SAS (side-angle-side), ASA (angle-side-angle) and RHS (right angle-hypotenuse-side) criteria. To perform this activity, the students can use papers kept in the lab or cut triangles with the help of scissors or cutters. They can also make use of Geo-Gebra software available on the computer in the lab. The lab, then, becomes a discussion room because of discussion on triangles drawn by the students at every stage of this game. Thus, the available material in the lab can be used to play games and do mathematical activities that helps in understanding different concepts through exploration.

The teacher may encourage the students to prepare similar models or charts using material, like thermocol, cardboard, etc., in the laboratory. The laboratory can also act as a forum for teachers to discuss and deliberate on some important mathematical issues and problems of the day. Using the laboratory equipment, the teachers can together device some games or activities or strategies to improve the understanding of mathematics in students.

A teacher can explain certain concepts using computers and calculators in the lab. Integrated projects that include other subjects, like science, social science, language etc., can be also thought of. Integrated projects help students to see a link of mathematics with other subjects, resulting in a better understanding of mathematics. Mathematics lab may not merely be restricted to mathematics teachers but can be of use to teachers of other subjects as well. They should also come and discuss integrating ideas of their respective subjects with mathematics.

The teachers and the students will be able to consult relevant reference books, journals, etc., in mathematics that are available in the laboratory. The students may be inspired by the works of great mathematicians through their pictures and information about them in the lab. The information about mathematicians (including Indians) may focus mainly on their works and struggles.

The students should be encouraged to use the materials kept in the lab and asked to present their observations about relevant concepts using those materials. A discussion among students about the presentations of their peers should also be encouraged. Arguments and counter-arguments may be encouraged. This will deepen the understanding of mathematics among the students and teachers. Based on these discussions, the students may be encouraged to improve the materials kept in the laboratory that would help in better understanding of the subject.

Thus, a mathematics lab can induce learners to handle concrete mathematical objects in a variety of ways, and gradually, lead them to handle the abstractions in mathematics.

Alex's Adventures in Numberland

Vandana Kerur*

Title: *Alex's Adventures in Numberland: Dispatches from the Wonderful World of Mathematics*
Author: Alex Bellos
Publisher: Bloomsbury Publishing India Private Limited
Language: English
Year of Publication: 2010
Price: ₹399

ABOUT THE AUTHOR

Alex Bellos is a British writer and broadcaster. He has a degree in mathematics and philosophy from the University of Oxford, and is currently a curator at the Museum of Science. He has authored books on mathematics, and also has an online coloumn in the *Gaurdian*. The book, *Alex's Adventures in Numberland,* is a bestseller, which was first published in 2010. It has been reprinted and translated into 20 languages.

REVIEW

Mathematics is a subject that is either dreaded or loved. In most students, a phobia or aversion or complete disinterest sets in early, with a rare chance of the attitude changing in later years. But there are also those who love the subject, and students who declare it as their favourite subject are the ones who can treat each problem as a puzzle, a challenge — solving it by applying the correct method, the exhilaration pushing them to take up bigger challenges.

Sadly though, neither type of student has been shown the fascinating side of the math world.

Ours is a result-oriented society. Of all disciplines taught in schools, mathematics lends itself to be a 'scoring' subject, where the correct application of a formulae can ensure 100 per cent marks. Eventually, the approach of teaching and learning

* *Cuemath Franchise Teacher Partner and Freelance Writer*, Goa

the subject has become restricted to mastering formula and applying them appropriately. Both teachers and students rarely pause to question, wonder and go beyond textbooks.

Bellos seeks to reconnect us with numbers around us. He decides to visit the world of maths, 'Numberland', as he terms it. As an adult, he can wander with a mind free of exam stress and relearn school mathematics with a fresh perspective and curiosity. Through anecdotes, he covers topics of school levels (Class V–XII), including arithmetic, algebra, geometry and statistics, presenting them in 12 chapters. The first chapter is interestingly labelled as Chapter Zero, since the topic discussed there is pre-mathematics, mainly about how numbers emerged. Chapters 1 to 11 take us away from the drudgery of problem-solving exercises to a world, where familiar terms and theories encountered in our texts are seen with a different eye. The Pythagoras Theorem, for instance, is revealed to be more than just a relationship between squares on the sides of a right-angled triangle. The area of a semicircle on the hypotenuse, for example, is equal to the sum of the areas of the semicircles on the other two sides. Amazingly, it holds good for any shape (regular or irregular) drawn on the three sides.

Bellos was curious about how different cultures and even religions approached mathematics, and how they helped shape it. He visits India, the land that gave the world 'zero', and learns about 'Vedic mathematics' among many other things. He explores Japan's love of abacus, origami and Sudoku. He tests his knowledge of probability at a Casino in Reno and attends the Mental Calculations World Cup in Germany. He travels around the globe, gaining a deeper insight as he moves on, and shares all of them in the book, weaving mathematical concepts with geography, culture, history and religion.

In his introduction to the book, he says it is aimed at the reader with no mathematical knowledge. This statement should not be interpreted to mean that it covers only anecdotes and trivia. On the contrary, it explains in detail proofs and explanations of all topics covered (several proofs of the Pythagoras Theorem are presented). We are introduced to mathematicians, their lives and works, their proofs and paradoxes. Concepts unheard of in textbooks are also discussed. Still, a person with no mathematical knowledge will not find the book tedious. The engaging style of the author and the stories presented will surely draw the reader deeper into the Numberland, where he/she can discover for himself/herself that maths need not be boring.

How is this book an asset if maintained in a school library? To do well in a subject, it is important to have an interest in it. This fact holds good both for teachers and the taught. A well-structured syllabus can provide students with a strong grasp of the fundamentals of critical thinking and problem-solving, but that is not enough. Educators need to realise that mathematics is not just about correct answers or derivations. If the teacher can understand and appreciate the complexity and beauty of the subject, he/she can pass on this zeal to the students, bringing out their ability to perceive and analyse. As each topic is taught in a class, a brief discussion on its real world application can make a huge difference to the level of interest generated. The Fibonacci Series, for example, is not just a sequence of numbers. It is abundantly seen in nature, in both the plant and the animal world. Origami is so much more than precise folding of paper. It has applications in robotics, creating heart stents and designing solar panels for satellites.

The book has around 400 pages and is well-structured. The content page includes a brief description of each chapter along with the page numbers. An introduction by the author is followed by 12 chapters. Each chapter is self-contained, meaning a topic is covered in its entirety with no continuing link from the previous chapter. This makes one free to choose and read the chapters in any order. Every idea presented comes with diagrams. A 16-page attachment in the centre of the book features coloured photographs of the places and people the author had visited during his travels. Detailed notes and appendices, a glossary of the mathematical terms used and an exhaustive index appear on the rear of the book.

To many students, mathematics has a stigma of being difficult and boring. Many teachers are of the opinion that it is a tedious subject to be taught. Bellos dispels both the myths and proves that it is a subject that can be taught without being intimidating.

International Mathematical Olympiad (IMO)

Ridhi Sharma*

Educational olympiads are organised across the world with an aim to bring forth the most gifted minds together to compete in various disciplines. Brilliant students are identified through regional and national competitions, and those who make it to the top are invited for a world-level competition.

One such unique olympiad is the International Mathematical Olympiad (IMO). It is the World Championship Mathematics Competition for high school students organised annually. The first IMO was held in Romania in 1959, with only seven participating countries. Since then, it has expanded, and today, about 100 countries participate in the anuual contest.

Logo of the IMO

In India, the initial phase of the Mathematics Olympiad is conducted through the Regional Mathematical Olympiad (RMO) at 25 centres. An examination for aspiring mathematicians of higher secondary level is held. It consists of six questions, which are to be solved in three hours. Thirty students are shortlisted from each centre. These 750 students attend the Indian National Mathematical Olympiad (INMO), which is conducted under the aegis of the National Board of Higher Mathematics (NBHM), in collaboration with the Homi Bhabha Centre for Science Education, which is the nodal centre. Of these 750 students, 30 are shortlisted to attend a one-month training programme at the Homi Bhabha Centre for Science Education, Mumbai. and then, six students are shortlisted on the basis of a variety of selection assessments. The programme aims at promoting excellence in mathematics among pre-university students.

* *Junior Project Fellow*, DEE, NCERT

At the IMO, each day students are given three difficult questions for seven points each, in up to two languages of their choice to be solved with in 4 hours 30 minutes daily for two days. The questions are framed by mathematicians of various countries. Obtaining one or two points out of 42 is appreciated by the panel. Bagging seven points for one question is treated as an extraordinary achievement. There have been instances, wherein the students have come up with a simpler solution to a question.

In 2016, the 57th IMO was hosted by the International Mathematical Olympiad, Hong Kong Committee Limited, on 11 and 12 July. A total of 608 students from across 100 countries and five continents took part in it. Six students from India participated in the Olympiad. Kapil Pause from Goa bagged the silver medal, while the others received bronze medals. It is laudable that none of the participants from India came without a medal.

The 58th IMO was hosted in Rio de Janerio, Brazil, from 12 to 23 July 2017. The Republic of Korea was declared as the winner. India ranked 52nd among 111 countries. India bagged 11 gold, 63 silver and 68 bronze medals, and 26 honourable mentions.

India has been participating in the IMO since 1989. Any number of attempts can be made in the IMO. Students can start participating in the olympiad from Class IV.

POST-SCRIPT

Unfortunately, even in this age of technology, there are some parts in of India where the awareness of IMO is negligible. Information about IMO should be disseminated by teachers and schools to students and their parents. Informal interactions in small groups on mathematics, including cyberspace, should also be encouraged. Besides, an attempt to open students' minds to the world of mathematics should be made.

REFERENCES

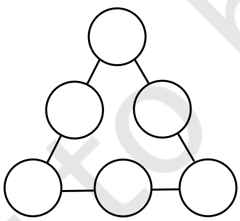
http://olympiads.hbcse.tifr.res.in/
http://www.imo2016.org/Home.php retrieved on 23/11/16
http://www.imo2017.org.br/
http://www.imo-official.org/
http://www.imo2018.org./index.html

Exploring Number Addition with Teachers

Dharam Prakash*

Exploration is one of the key activities in a classroom, which can generate interest and initiate learning about particular concepts or theme. There are many questions in the minds of teachers, who are willing to initiate their students into exploration. Some of the questions are: "How does one create an exploration activity?", "what is an exploration activity?", and "when and how does one know where to stop?"

To give primary teachers, teaching Class I and II mathematics, an experience of exploration about numbers and number operations, it was planned to give them the following exploration assignment:



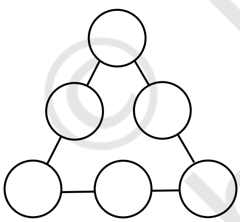
In this picture, six circles have to be placed in such a way that each side of the triangle adds up to the same number (9). For a trial, this assignment with the picture on a sheet of paper was given to one teacher to solve. It was being observed how the teacher was exploring or making attempts to solve it. The teacher first wrote some numbers in the circles and tried to add them but it did not work. Then, she erased all the numbers and started writing the numbers again. After two-three attempts, she looked around and said, "It looks a simple challenge but is difficult." After two–three more such attempts, she was able to do it and finished with a smile.

Observing the person doing the exploration task made one think: "What can we do to make this exploration task interesting? How do we introduce the fun and challenge element into this task? Is it possible to move the numbers to avoid writing and erasing?"

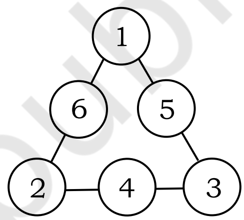
* *Professor (Retd.),* ESMP, Department of Elementary Education, NCERT

The last question triggered further questions. “How can one make the numbers movable to introduce fun element into the task?” There were many suggestions about it. Finally, circular plastic counters (which are used in playing ludo or board games) were chosen. It was also decided that each participant teacher would create his/her own number counters for the task by writing the number counter for the task and write from numbers 1 to 9 on the counters. Afterwards, it was also worked out how the activities for exploration would be taken up further.

On the scheduled day, an the interaction with the teachers of classes I and II started. Each teacher was provided with 10 plastic counters and asked to write numbers 1 to 9 on them (leaving one counter blank for later use). Then, the picture as shown below was drawn on the board and the teacher participants were asked to draw the same on a sheet of paper.



Once this was done, the group was ready for exploration. The exploration task was announced — place the numbers (written on the counters) in such a way on this figure that the sum of each side is 9. A remark was overheard “Oh! It seems easy!” But when each one started moving around the counters (with numbers written on them), the fun began. It was observed that most of the teachers were trying different positions of numbers and adding them to check whether the sum was 9 or not. Remarks, like “does this task have a solution?”, were also heard. Interestingly, not only the participant teachers were exploring and making their own strategies but also commenting on the strategies of other participants. However, some teachers were able to find a solution. But there were some who were still trying. The discussion on the solution started with one teacher writing the solution on the board.



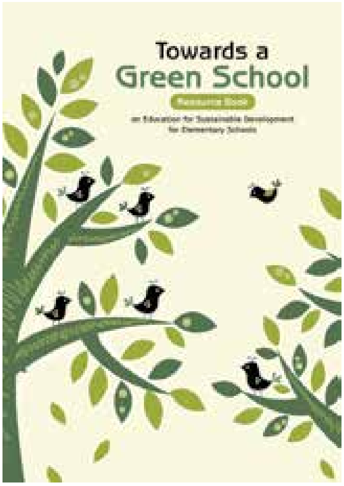
In response to the question “how did you get to the solutions? Please recall”. The teachers came up with their exploration efforts and explained different permutation combination they had tried to get the solution. It was interesting to note their enthusiasm for exploration. It was obvious that they were ready for more exploration and the next assignment was — “can you rearrange these numbers in such a way that sum of each side is 11? The exploration about single-digit numbers and their addition had begun in the real sense.

Later, the teachers expressed that they had understood what exploration was all about. The fun, challenge and joy of discovery that they had experienced enhanced their learning and insight about single-digit addition.

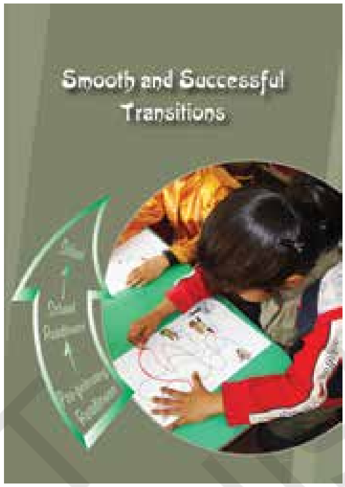
I request the readers to involve themselves into exploration activity and try to find out how to arrange numbers 1 to 6 in the figure given in the article in such a manner so that the sum of numbers on each side is 12.

Do write to us about your experiences of exploration about numbers.

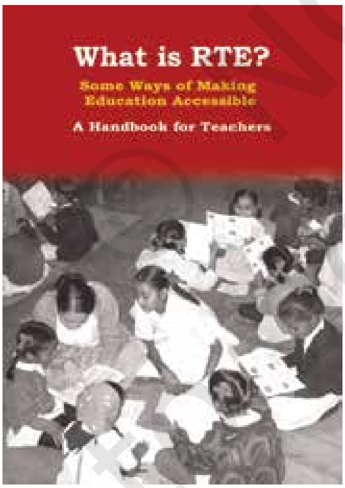
Some other NCERT Publications



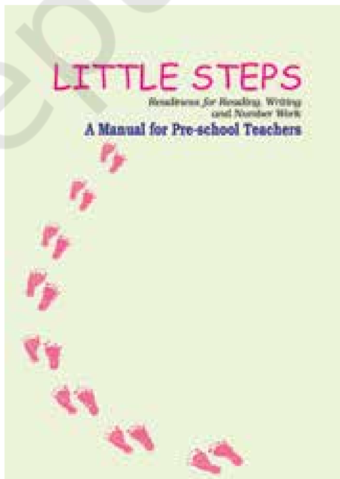
₹ 115.00 / pp. 170
Code – 13155
ISBN 978-93-5007-835-8



₹ 65.00 / pp. 128
Code – 32115
ISBN 978-93-5007-821-1



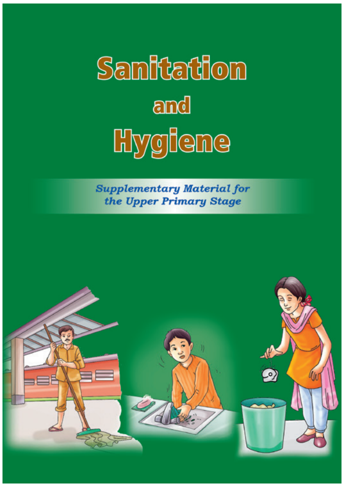
₹ 90.00 / pp. 148
Code – 13105
ISBN 978-93-5007-276-9



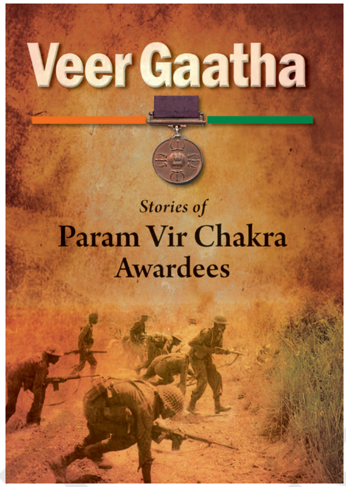
₹ 100.00 / pp. 126
Code – 1556
ISBN 81-7450-335-8

For further enquiries, please visit www.ncert.nic.in or contact the Business Managers at the addresses of the regional centres given on the copyright page.

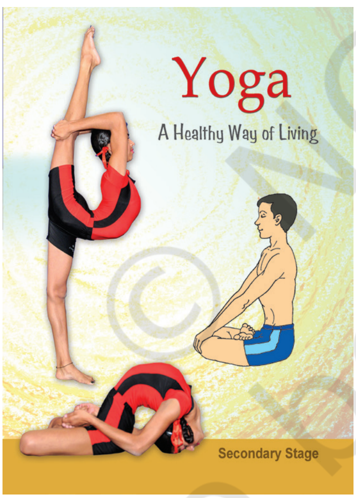
Some other NCERT Publications



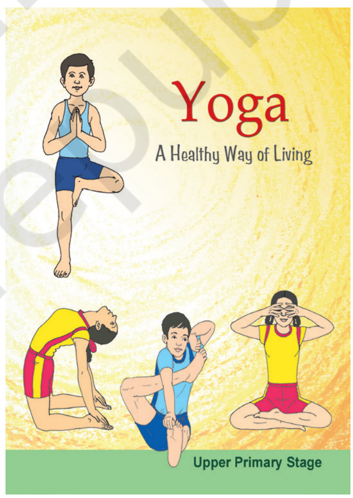
₹ 40.00 / pp. 40
Code – 18148
ISBN 978-93-5007-857-0



₹ 100.00 / pp. 140
Code – 18139
ISBN 978-93-5007-765-8



₹ 50.00 / pp. 80
Code – 13128
ISBN 978-93-5007-344-5



₹ 65.00 / pp. 100
Code – 13127
ISBN 978-93-5007-345-2

For further enquiries, please visit www.ncert.nic.in or contact the Business Managers at the addresses of the regional centres given on the copyright page.

Regd. No. 28935/76

विद्यया ऽमृतमश्नुते

एन सी ई आर टी
NCERT

राष्ट्रीय शैक्षिक अनुसंधान और प्रशिक्षण परिषद्
NATIONAL COUNCIL OF EDUCATIONAL RESEARCH AND TRAINING