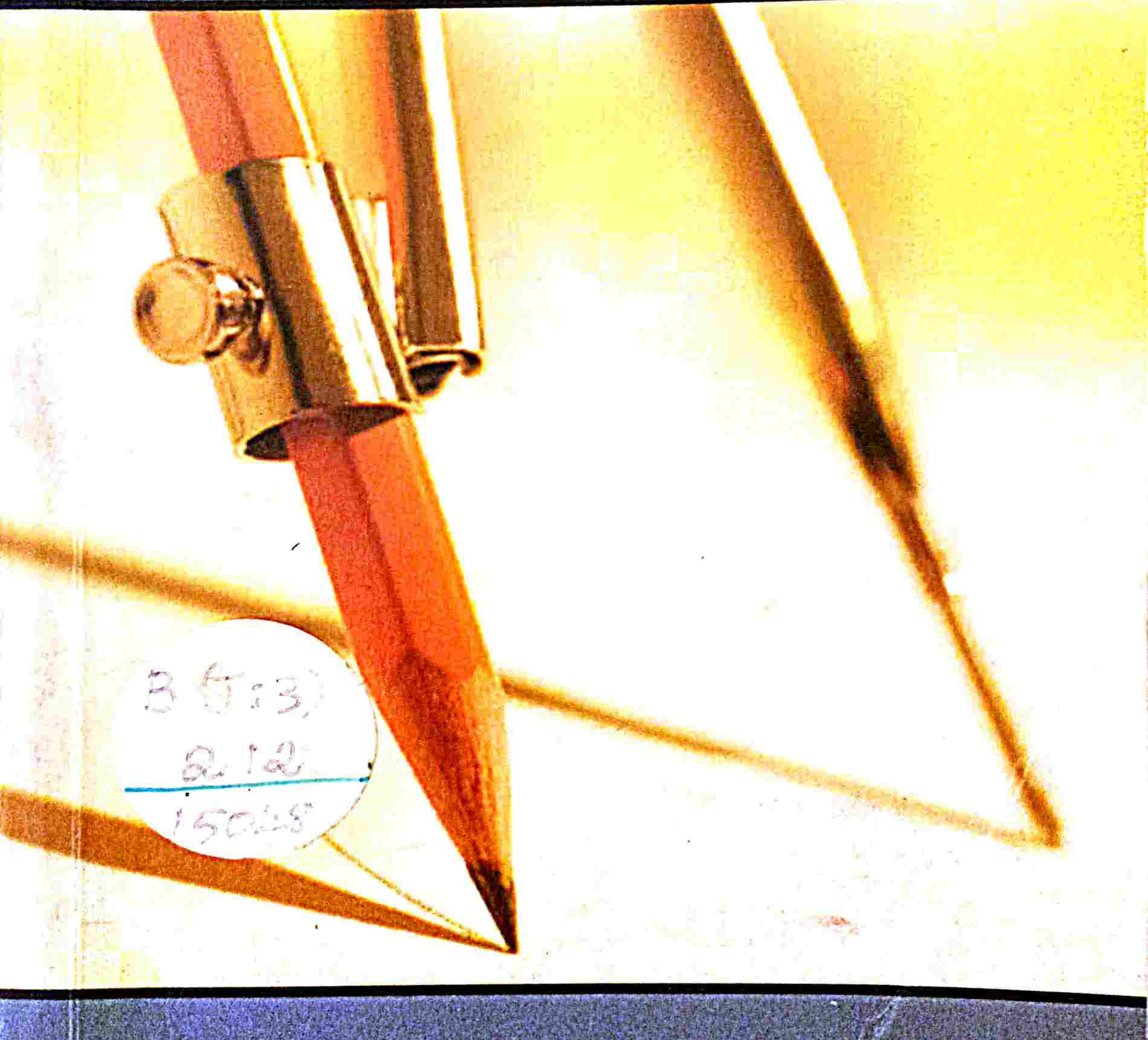
Becoming A Reflective Mathematics Teacher



Dr. B. C. Sobha

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Dr. B.C. Sobha
Ph.D.

With Compliments

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PREFACE

The teacher who is to direct learning of Mathematics effectively requires thorough knowledge of subject matter - both traditional and modern Mathematics. She/he must understand how pupils learn Mathematics and how numerous techniques and devices can be used for promoting learning. To capture pupils interest and to help them acquire the necessary skills is the first concern of the Mathematics teacher. Change in Mathematics curricula raised further demands on the teacher. The key to developing into the type of Mathematics teacher who can empower students with the love and understanding of Mathematics is to continually explore ways that lead to sustainable changes in both thinking and class-room practice.

This book is designed to help teachers in promoting effective teaching-learning process in Mathematics. Those, whose task is to direct or carry on the actual class-room instruction would be concerned not only with the general consideration of the curriculum, but also with many instructional problems arising from the subject matter itself and with the specific difficulties which the students often encounter in the study of Mathematics. This book does cater to this problem of teachers. Great teachers are not born, they are made. Just as the most talented musicians or artists become great by reflecting on their art, beginning teachers become accomplished teachers, and skilled teachers become great teachers, by thinking hard about their teaching and finding ways to improve it. Sincere efforts have also been made to detail prominent views of renowned Mathematicians, Educationalists, Scientists and others related to the field of teaching of Mathematics.

ACKNOWLEDGEMENTS

Behind every book there is a cast of characters making a unique and essential contribution. Here the cast ranges from my wise and insightful teachers, Principal of our college, encouraging Management committee members of N.V.K.S.D College of Education, colleagues, students, friends and supportive family members. As it is impossible to thank every body individually I would like to take this opportunity to acknowledge their collective, informative influence on my thinking.

I acknowledge with gratitude the effort taken by WorldBooks in bringing out this book.

Sobha. B.C

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CHAPTER - 1

HISTORY OF MATHEMATICIANS AND MATHEMATICS

"Mathematics is the mirror of civilization". There is no exaggeration in this saying because history of mathematics is the history of civilization. Mathematics represents a high level of abstraction attained by the human mind. The historical back ground of development sequence of mathematics has been found in the studies and research of tribal language and extinct language related to them.

The history of mathematics depicts the different stages in the development of mathematics through out the ages. It shows that mathematics has been developed in the frame work of well developed urban civilizations and well organized economic conditions.

In India mathematics has its roots in vedic literature which is nearly 4000 years old. In ancient age, Babylonian, Egyptian and Greek Civilization contributed to a great extent in the development of mathematics. In comparison to middle age, mathematics developed more rapidly in ancient and modern age.

In ancient times man used various methods to count. It is believed that lines traced on the ground, cuts in the trees, pebbles, notches and fingers etc, were used to count. The notation system developed differently in different countries. The Babylonians used wedge shaped symbols. A vertical wedge 'V' stood for number one. The ten was represented by 'L' and hundred by 'VL'.

The roman system of notation might have developed by tracing a vertical line for one, two lines for two and three lines for three. V probably represented the five in the form of whole hand. The line before and after the symbol 'V" were taken to represent a number smaller by one and greater by one respectively. The symbol X perhaps was taken as the combination of two fives. Thus the roman system of notation took the form as I, II, III, IV, V, VI, VII, VIII, IX, X and so on.

The Concept of Zero

The invention of zero is the most valuable contribution of ancient Indian thinkers to mathematics. If zero and decimal system were not invented it was not possible to present and write the large numbers.

Exactly it is not known, when zero was invented and who invented it. But, there are various examples of its use in vedic period. The progress of mathematics and science is possible due to the invention of zero. In ancient India the terms used to describe zero included Pujyam, Shunyam. Bindu the concept of a void or blank was termed as shukla and shubra. The Arabs refer to the zero as siphra or sifr from which we have the English term Chiper or Cipher. In English the term cipher connotes zero or any Arabic numeral. Thus it is evident that the term cipher is derived from the Arabic sifr which in turn is quite close the Sanskrit term Subhra.

Contribution of Mathematicians

The world has produced a galaxy of mathematicians in its past history and in our present days a number of such persons exist who have contributed one thing or the other in this field of knowledge.

Arya Bhatta (476 A.D.)

Arya Batta was born at Pataliputra near Patna in Bihar about 476 A.D.. He mentioned his own date of birth, in his book "Arya Bhatia" dealing with astronomy and mathematics.

He gave the idea of representing unknown quantities with letters. The digits from 1 to 25 are represented by the first 25 letters for example, Ka (keA) means 1, Kha (Ke) means 2 and so on. Where as De, Dee F...etc. represented 10^0, 10^1, 10^2, 10^3... and so on.

In his book he had given 121 sholkas divided into four parts (i) Gati–Paadika (ii) Ganit– Paadika, (iii) Kaal Kriyapad and (iv) Ga– Pad. He was the first person to present Arithmetic, Algebra and Geometry in his astrological calculations. He suggested numerous rules. Besides this he gave the idea of decimal system, properties of similar triangle, method of finding sum of squares and cubes of natural number and Pythagoras theorem.

To sum up Arya Bhatta is really one of the greatest genius of his time in the field of mathematics and astronomy.

Brahma Gupta (628 A.D.)

Brahma Gupta was born in 628 A.D. at Punjab. His father was Vishnu Gupta, he lived at ujjain and worked in great astrological laboratory of Seventh Century established at Ujjain. He wrote his first book Brahm-sphuta Siddhantha, which contains great knowledge of arithmetic, algebra, geometry and astronomy in its 21st chapter. He has used the value of pie $\left(\pi = \sqrt{10}\right)$. He also invented four different methods of multiplication viz. Gan Mutrika, 'Khanda', 'Bheda' and Ista. He has given a detailed account of progression, areas of triangles, quadrilaterals, slopes, volumes of trenches and amount of grains in heaps etc. His work on arithmetic includes integers, fractions, progressions, simple interest, the mensuration of plain figures and problems on volume.

He gave the exact concept of zero by defining it as (a-a)= 0. He gave the rules to deal with negative number as under.

I. Negative multiplied or divided by negative becomes positive

II. Negative subtracted from zero is also positive.

He was also interested in solving indeterminable equation. He was the first mathematician who applied algebra to astronomy.

Mahaviracharya (850 A.D.)

Mahavirachaya was the follower of Jain religion. Very few literature is available about his life history. It is assumed that he was born in about 850 A.D. in Mysore. He is known for his famous book 'Ganita Saar Sangrah', 'Shatrinshika' and 'Jyotish Patal'. The book 'Ganita Saar Sangrah' has been translated into English by Dr. E. Smith. According to Smith, the analysis of oral and written questions on trigonometry given by Mahaviracharya is far superior to that given by Brahmagupta and Bhaskara.

In 'Ganita Saar Sangrah' the process of arithmetic has been explained in detail. Very interesting problems have been included in his collection. He has also dealt with conversion of rectangles into squares and vice-versa, of circles into squares and vice-versa. He has also given details of various kinds of triangles. He considered mathematics to be the most useful subject that is used in all fields of knowledge. He also studied the different properties of ellipse.

Sridharacharya (850 A.D.)

Sridharacharya was a follower of Shaiva religion and later changed over to Jainism. He was born in about 850 A.D Thus, he was contemporary to Mahaviracharya. He also belonged to Mysore. In his childhood he was a scholar of Sanskrit. Later on he studied mathematics and wrote the books 'Nev-Shatika', 'Trin-Shatika', Paati-Ganita' and 'Beej-Ganita'. His works include treatise on square root, cube root, fractions, compound practice, progressions, areas and interest.

His book 'Pati Ganita' has been translated into Arabian language entitled as 'Sarala Tarabt'.

Arya Bhata-II (950 A.D.)

Aryabhata II was born in Maharastra in 950 A.D. He complied a book 'Maha Arya Sidhanta'. In this book the operations of arithmetic and solution of linear indeterminate equations (Kutak) has been explained. He has also dealt with surface and volume of sphere to a great extent of correctness. He has given the value of pie (π) - 22/7, which has been accepted by all.

Bhaskaracharya-II (1114 A.D.)

In the history of Indian mathematics, the name of Bhaskaracharya II is commonly taken along with notable mathematician Aryabhatta I. He was born in 1114 A.D. at some place near Bijapur in Hyderabad. His father was a learned man in Vedas and Shastras. Bhaskaracharya has a very keen interest in Astrology. He served as head of the astrological laboratory at Ujjain. He is well known for his treatment of negative numbers, which he considered as debts or losses and also for his treatise on arithmetic and measurement which he had named for his daughter Lilavati.

He complied several books on different branches of mathematics. Some of his famous books are 'Shiddhanta Shiromani', 'Karam Kantoohal', 'Samay Siddhanta Shiromani' and 'Surya Siddhanta'. His no-table work is complied in 'Siddhanta Shiromani', which has four parts and each part is sub-divided into several chapters. The first part of this book is named on behalf of his daughter Lilavati. There is a tragic story behind the naming of the book.

"Bhaskara, who was an astrologer as well as a mathematician, has discovered the propitious day, hour and moment for the marriage of his daughter Lilavati. Any other time was prophesied as being sure to bring misfortune. For knowing the actual good time, a cup with a thin needle hole was put swimming in a large pot with full of water. While the bride was watching the cup, whose sinking would mark the proper moment, a pearl dropped from her head dress and stopped the hole in the cup. The lucky moment passed without sinking the cup. Thus, Lilavati remained unmarried. Bhaskara consoled her daughter and in compensation promised her that he would give her name to a book which should last until the latest time."

In this book 'Lilavati', questions have been composed in a very interesting manner. It includes notation, the operations with integers and fractions, the rule of three, interest, series, most common commercial rules, allegation, permutations, mensuration and simple algebra. It also includes rules related to zero viz. $a+0 = 0$, a multiplied by zero equals to zero or $a.0=0$, powers of zero are one. He gave the idea of infinity by dividing a number by zero. i.e, $1/0=\infty$.

He also gave many important formulas for measurement, example.

Volume of sphere = Area of sphere x 1/6 of its diameter.

His book 'Beej Ganita' is a book on Algebra. In this book he discussed directed numbers, the negatives being designated as debt or loss and being indicated by a dot over each number and usual rules and formula being stated correctly. He dismissed the imaginary numbers with the statement.

"There is no square root of negative quantity for it is not a square."

In 'Sidhanta Shiromani', he dealt with astronomy and has asserted the sphericity of the earth.

He also gave proof of Pythagoras theorem. He gave a method of deducting new sets of solutions of $X^2 + 1= Y^2$ from one set found by trial. His works on equation is certainly the finest contribution in the theory of numbers before Lagrange. So his place is very high in the domain of mathematicians. In a Journal of Royal Society, Dr. Stopwood has remarked, "we must acknowledge the intimate capabilities of Bhaskaracharya."

He also had the knowledge of gravitational power of earth. He has mentioned it in one of his books.

Modern Age (After 180 A.D.)

In 19th century Nrisingh Bapu Dev Shashtri and Sudhakar Dwivedi were the eminent Indian mathematicians Dev Shashtri compiled a book on Indian and western mathematics. Sudhakar Dwivedi wrote many books on the properties of ellipse, spherical trigonometry, calculus etc.

The names of Dr. Ganesh Prasad, Srinivas Ramanujan, and Swami Bharti Krishnatirtha can rightly be taken as eminent mathematicians of present era.

Ganesh Prasad

Dr. Ganesh Prasad was born on 15th November, 1876 AD in Ballia, Uttar Pradesh. His elementary education was completed in Ballia city. He completed his M.A. degree in Mathematics from University of Allahabad. He also obtained D. Phil and Doctor of Science in Mathematics. He was first Indian who got the degree of Doctor of Science in Mathematics. One of his most valuable contribution in the field of educational development was to establish Agra University in 1927. He wrote many valuable books and published several research papers. His first research paper on 'Elliptical functions and Spherical Harmonics' was published in the journal 'Messenger of Mathematics'. He also detected and corrected the errors in the works of eminent French Mathematician Lebesgue.

He remained in European countries for about five years. He was very popular among students and teachers of Cambridge University. After returning from Europe, he was appointed as Professor of Mathematics at Allahabad University. He died on 9th March, 1935 at Agra after attending the meeting of University council.

Sriniwas Ramanujan

Sriniwas Ramanujan Aiyanger (briefly know as S. Ramanujan) was born on December 22, 1887 at a village Erod of Tanjore district in Tamilnadu state. His father was a 'munim' to a cloth merchant at Kumbakonam. He

passed the high school examination at the age of 16 from Town High School Kumbakonam.

Ramanujan was interested in mathematical studies from elementary stage of education. He entertained his friends with theorems and formula and the square root of two to any number of decimal place. Thus, he surprised his teachers.

His greatest contribution to the world of mathematics was in the field of theory of numbers. In fact he had a very original and intuitive approach to numbers. He remembered the idiosyncrasies of every of the first ten thousand integers to an extent that each one of them became his personal friend. While doing the research work with Professor Hardy, a great mathematician of England, he invented many remarkable and interesting facts about numbers.

When he was ill in England, Prof.Hardy went to see him. While talking with him Prof.Hardy said that he came by a taxi which number was 1729. It is a bad number because it is divisible by 13 and 19.

$1729 = 7 \times 13 \times 19$

Ramanujan said "'No', it is a very interesting number." It is that smallest number which- can be presented in the form of sum of two cubes in two ways.

$1729 = 12^3 + 1^3 = 10^3 + 9^3$

Prof. Hardy became very happy with this answer and remarked about his work on numbers.

Ramanujan studied the highly composite numbers, their structure; distribution and special forms. He presented many statements about numbers. For example, he made the statement that every even integer greater than two is the sum of two primes. Thus, 4 is the sum of two primes 1 and 3, 6 is the sum of two primes 3 and 3 and 8, is the sum of two primes 3 and 5 and so on.

The another problem which engaged his attention was partition of whole numbers. He found that there are certain number of ways to part a whole number; if we use only integer. Take the case of number 3. There are three alternative ways to write it. 3+ 0,1 +2, 1+ 1+1. Similarly, 4 can be written in five different ways.

4=4+0=3+1=2+2=2+1+1=1+1+1+1

It may be easily verified that there are no other ways of partitioning the above numbers, if we do not wish to use fractional numbers. Although this problem of partitioning any number into its competent integers has proved to be very difficult but he developed a formula that is valid for any number.

He also contributed in the field of higher geometric series and continued fractions. He found quite a number of results in the fields of definite integrals and elliptic functions. He also investigated about divergent series.

He was honoured by England in 1918 by electing him as Fellow of Royal Society. When he returned to India Prof. GH. Hardy remarked, "Ramanujan is returning to India as the greatest mathematician India has ever produced. I hope his country will give him a befitting reception". Although he could not get honour in India for which he deserved in his life, every Indian is proud of this great genius of the country.

The cruel hand of death snatched him away too early at the age of only 37 in 1920. But the name and contribution of this great Indian mathematician will remain immortal in the world of mathematics.

Eminent Mathematicians of Western Countries

There is a long list of mathematicians whose dedication and sustained work put mathematics at present height. Some of them have become immortal because of their remarkable contribution to mathematics. They will be remembered by mankind for all times to come and the world will remain indebted to them for their great contributions.

Pythagoras (580-568 B.C.)

The exact date and place of birth of this great mathematician are unknown. Some evidences indicate that he was born between 580 and 568 B.C. It is generally believed that Pythagoras came from the Island of Samos belonging to Joian colony of Greeks planted on western shore and Island of Asia Minor. But, there is no doubt that he lived in Greece at the time of dawn of the golden age.

Pythagoras was the pupil of great mathematician Thales (640 B.C - 550 B.C) After studying under his great teacher he travelled to Egypt to gather knowledge. Possibly he may even have come to India. He settled in Crotona, a Greek town in South Italy, where he gathered a group of wealthy persons and founded the brotherhood of the Pythagoreans. He used to deliver lectures which were so great, that they willingly formed themselves into a society called 'order of Pythagoreans'. This influence was more religious than political. They bound themselves with an oath that they would not reveal the secrets and teaching of the school. A 'star' shaped pentagram was the distrinctive badge of brotherhood. His brotherhood has ever since served as a model for all secret societies in Europe and America. Pythagorean Philosophy placed great importance on the idea of the infinite and taught the doctrine of transmigration of souls. Their astronomy, had advanced to the point of considering that the earth as a sphere. Pythagoras based his philosophy upon the postulate that number is the cause of the various qualities of matter. This led him to exact arithmetic, as distinguished from logistic and to consider it as one of the four degrees of wisdom - arithmetic, music, geometry and astronomy. He asserted that unity is the essence of number, the origin of all things and the divine. Thus, he had the idea of limited and unlimited which led to form the idea of time, space and motion.

He is credited to have discovered the proof of the theorem of right angled triangle. To Pythagoras, numbers were not attributes, but they were, the stuff out of which all objects, we see are made - the rational reality. He identified 'one' with the point, 'two' with the line, 'three' with the surface and 'four' with the solid. He attributed various numbers and form to physical elements. For example, 'five' is the cause of colour, 'six' of cold, 'seven' of health and 'eight' of love. Another, kind of problem which interested Pythagoras was 'application-of areas'. He introduced the term 'ellipse', 'parabola' and 'hyperbola'. He made substantial progress in the theoretical side of arithmetic and algebra. He also studied various types of progression - arithmetic, geometric and harmonic.

He is also credited for inventing the musical science or the harmonic canon. He is said to have discovered the -harmonical progression in the notes of musical scale, by finding the relation between the length of a string and the pitch of vibrating notes. He seems to have held that the intervals between the heavenly bodies were determined by the laws of musical harmony and hence arise the doctrine of the harmony of the spheres. He had great

interest in the properties of numbers. He devoted considerable attention to the study of areas, volumes, proportions and solids. The followers of Pythagoras tended to attribute their own works to their master. They preserved the teaching of their master and simplified his discoveries. Many great scholars of the time like Plato and Zeno studied under the Pythagoreans.

The Pythagoreans made substantial progress in the theoretical side of arithmetic and algebra. They had a geometrical equivalent for our method of solving quardratic equations. Aristotle has remarked, "The Pythagoreans first applied themselves to mathematics, a science, which they improved and penetrated with it, they fancied that the principles of mathematics were the principles of all things."

Pythagoras died in 500 B.C away from his country in exile for political reasons but very soon world realized his greatness. Later on government of Italy erected his statue in Rome to pay honour to 'the wisest and bravest of the Greeks'.

Plato (427-347 B.C.)

The great philosopher, political thinker, mathematician, social reformer and educationist Plato was born in 427 B.C in Athens. At the age of 20, he came in contact of another great philosopher of Athens, Socrates and had studied for eight year under his guidance. Plato exerted a great influence on mathematics by founding an academy in Athens. He contributed to mathematics in general and geometry in particular. It is evident from many mathematical illustrations in his books. He was so enthusiastic and influenced by mathematics that he has put an inscription on his lecture room of academy which read as "Let no one destitute of Geometry enter my doors".

He believed that a man should be trained to see below the surface of things. He had given great importance to mathematics and particularly to geometry by replaying the question "What does God do?" with the statement, "God always geometrize".

Euclid (330 B.C.—295 B.C.)

Euclid, a well known name to every student of mathematics has the distinction of being the only man to summarize all the mathematical knowledge of his times. Nothing can be said with certainty about the life history of Euclid. Most possibly, he is an Egyptian and not a Greek. But who he was has very little importance in comparison to his contribution to geometry. Ptolemy, the successor of Alexander of Macedonia, the first conqueror of the world established a university at Alexandria, a city which as the meeting place of Greeks, Jews and Arbs. Euclid worked as a teacher to this university for about 30 years. The ancient mathematics was perfected and treasured in the library of this university and other libraries of the city.

Euclid wrote several books. His most important work is 'Element' a collection of thirteen books.

In Euclid's books the concepts were defined at the beginning, then postulates were stated and also those generally accepted hypothesis which we call axioms but which Euclid called. "Common notions". His 'Elements' began with with construction problems. Although his work lacked motivation, yet it has been the most widely used text that the world has ever produced. The popularity of Euclid's Elements is evidenced by the fact that it was translated from Greeks to various other languages. Many other writings of Euclid included his dealing with astronomy, music and optics.

Napier (1550 A.D.—1617 A.D.)

John Napier belonged to noble Scottish family. He acquired great reputation as an inventor. He made a sort of chart-arithmetic in which digit moved rooks and bishops on board. He propounded the binary system of writing the numbers as against decimal system.

In 1950 A.D., Napier discovered Logarithm - a device which replaces multiplication by addition. He prepared the tables of logarithm which proved very practical in other branches of mathematics and sciences. The preparation of tables of logarithm was a life -long task. He dedicated 25 years in preparing the tables and then it could be published. All this was done before either the theory of indices or differential calculus had been invented. Napier's treatment of the subject shows intimate knowledge of correspondence between arithmetical and geometrical progression. He was also a geometer and devised new methods in spherical astronomy.

Gauss (1777 A.D.—1855 A.D.)

The great German mathematician Karl Fredrick Gauss was born in a poor family. His father was a mason labourer and wanted to make his son like him. But his mother was very aspirant of her son. Once she asked to a friend of Gauss, "what is your view about the future of Gauss?" He replied at once, 'greatest mathematician of Europe' and undoubtedly he was quite right.

The marvelous aptitude of Gauss for calculation at early age attracted the attention of his teachers who brought him under the notice of Charles Williams, Duke of Brunswick (Germany). The Duke (King) undertook to educate this brilliant boy and sent him to the collegium Corolinum. He had a high aptitude in mathematics as well as in language, he began jotting down in a copy book very brief Latin memoranda of his mathematical discoveries. This diary was published in 1901. The first entry of this diary dated March 30, 1796, refers to his discovery of a method of inscribing in a circle a regular polygon of 17 sides. This discovery encouraged him to pursue mathematics. He made several discoveries as a student at Gottingen. He was appointed as astronomer in the newly opened observatory at Gottingen.

He contributed substantially to every leading field of mathematics. His diary reveals pioneer facts, higher trigonometry and elliptic functions. He also developed the theory of surfaces with special attention to their curvature and conditions for one surface to fit another. The method of 'least squares' which is indispensable at present was invented by Gauss. In 1909, he published his famous book 'Theory of Motion of heavenly bodies besides revolving round the sun in conic section'. He also made many significant advances in Geometry.

Gauss is usually rated with Archimedes and Newton as one of the three great mathematicians. Kronecker said of him that "almost everything which the mathematics of our country has brought forth in the way of original scientific ideas is connected with the name of Gauss."

Meaning And Nature of Mathematics

Mathematics plays a vital role in the day to day life. It is a very important subject. Therefore before imparting and transmitting its knowledge it is necessary to understand 'what is mathematics? What is its nature etc? There are various definitions of mathematics. The term mathematics has been interpreted and explained in various ways. Mathematics deals with the quantitative facts and relationships as well as with problems involving space and form. It enables man to study various

phenomenon in space and establish different type of relationship between them. There fore it may be concluded that mathematics is the enumerative and calculative part of human life and knowledge. It helps the person to give an exact interpretation to his ideas and to reach on certain conclusion.

Definitions of Mathematics

In Oriya mathematics means 'GANITA' meaning Science of Calculation. According to Oxford dictionary - "Mathematics is the Science of measurement, quantity and magnitude".

According to Webster's dictionary 'Mathematics is the science of number and operations like inter relations, combinations, generalizations and abstraction and of space, configurations and generalizations."

In the words of Bertrand Russel "Mathematics may be defined as the subject in which we never know what we are talking about nor whether what we are saying is true".

According to Prof. Voss. "Our entire civilization depending on the intellectual penetration and utilization of nature has its real foundation in the mathematical science".

According to Locke "Mathematics is a way to settle in the mind of children a habit of reasoning".

On the basis of above definition it can be concluded that.

- Mathematics is the science of space and number.
- Mathematics is the science of calculation.
- Mathematics is the science of measurement, quantity and magnitude.
- Mathematics is a systematized, organized and exact branch of science.

It deals with quantitative facts and relationship.
- It is the abstract form of science.
- It is the science of logical reasoning.
- It is an inductive and experimental science.

Why We Study Mathematics?

The knowledge of fundamental process of mathematics and the skill to use them are the preliminary requirement of human being in any society of modern time.

One cannot lead his daily life activities very well without basic knowledge of mathematics. A common man can do well various activities without learning how to read and write but he can never pull on without learning how to count and calculate. A person may belong to any kind and class of society but, he utilizes knowledge of mathematics in one form or another. So there is unavoidable practical utility of mathematics in every one's life.

Therefore it is necessary for every person to have a basic knowledge of mathematics to lead his daily life activities properly. The knowledge and skill in the fundamental process of mathematics can be achieved through the systematic study of this subject. Besides the above practical utility of mathematics in daily life, its study is also essential due to following reasons.

- Most of the occupations, by which the needs of peoples are fulfilled cannot run without the use of mathematics.
- The entire business and commercial system is the best of the knowledge of mathematics.
- It is helpful in the study of other sciences.
- It is indispensable instrument for all physical research.

- The progress of a nation depends upon the progress of mathematics.
- It is useful in budgeting.
- It is correlated with other subject.

So mathematics occupies a prominent place in our life . That is why its study is indispensable for us.

Nature of Mathematics

Mathematics is the gate way of all sciences. In school those subject which are included in the curriculum must have certain aims and objectives on the basis of which its nature is decided. Mathematics holds a strong and unbreakable position as compared to other school subject. Mathematical predications are almost accurate and hundred percent true. The nature of Mathematics is enlisted as follows.

- Mathematics is an exact science. Mathematical knowledge is always clear, logical and systematic and that may be understood easily.
- It is the science of space, numbers, magnitude and measurement.
- Mathematics involves conversion of abstract concepts into concrete form.
- It is the science of logical reasoning.
- It helps the man to give exact interpretation to his ideas and conclusion.
- Mathematics propositions are based on postulates and axioms from our observations. In physical and social environment we form certain intuitive ideas or notions called postulates and axioms.
- It convert abstract phenomenon into concrete. Thus abstract concepts may be explained and understood with the help of mathematics.

- It is related with each aspect of human life.
- Mathematical knowledge is developed by our sense organs therefore it is exact and reliable.
- The knowledge of Mathematics remains same in the whole universe, every where and every time. It is not changeable.
- The knowledge of mathematics has no doubt. It provides clear and exact response like yes or no, right or wrong.
- It involves inductive and deductive reasoning and can generalize universally.
- It helps in self evaluation.

Values of Teaching Mathematics

Mathematics is considered as father of all sciences. It helps the students in achieving the educational goals and objectives. The importance of mathematics can be expressed in the form of values. It has a practical value in life. So it is called the Practical cultural science. Apart from the personal life mathematics has an importance in social life. Mathematics has cultural value and disciplinary value also. Briefly, the following values can be derived through the teaching of mathematics.

A. Utilitarian Value

Mathematics apart from the value of bread and butter has a value of wider practical applications. Principles and theories of mathematics are applied in different aspects of life. Thinking and reasoning is very much guided by mathematics and it is this thinking and reasoning that help in building the society.

Mathematics is helpful in other branches of science such as Engineering, Physics, and Economics etc. No business can flourish without practical knowledge of

Mathematics. The teaching of mathematics is therefore guided by this utilitarian aim.

According to Roger Bacon- "Mathematics is the gate and key to all sciences." Our whole universe is charged with mathematics. Every occupation in this world requires mathematics at every state. We need mathematics in order to adjust our expenditure to income. According to Kent- "A Science is exact only in so far as it uses mathematics".

So mathematics is responsible for giving us a system, organization and essential abilities for leading a successful life. We shall remain too much handicapped in our life in case we remain ignorant of mathematics.

B. Social value

The change in the social structure with regards to modern facilities like mode of transport, means of communication and progress in the field of sciences and technology is due to mathematics only. Ideal education is that which helps to make a child a qualified and useful citizen of society from the beginning.

Napoleon has accepted the social value of mathematics and said, "The progress and improvement of mathematics are linked to the prosperity of the state".

In this way mathematics plays an important role in not only understanding the progress of society but also to develop the society. At present our social structure seems to be so scientific and systematic and its credit goes to mathematics. In its deficiency, the entire social system will be disturbed.

C. Cultural Value

The culture of every nation or society has its unique characteristics. It has its own importance. Each nation or society reflects its culture by its living standards, rituals, artistic progress, economic, social and political aspects etc. The history of mathematics presents the image of culture of different nations. A person is said to be cultured if he is well educated and have refined manners of dealing. The person becomes critical observer, logical thinker and proper knowledge of mathematics changes the mind of the person. Thus the person becomes more cultured with the proper knowledge of mathematics. The famous mathematician Hogben has remarked that "Mathematics is the mirror of civilization". In fact mathematical knowledge is indispensable and changes the way of ones living.

Mathematics not only make us familiar with culture and civilization but also helps in preserving and promoting cultural heritage and transmitting it to future generations. Through the application of scientific and mathematical discoveries our culture and civilization is undergoing constant change. The welfare of our civilization is now almost wholly dependent upon scientific as well as mathematical progress. It affects our view of life and way of living as a result of which it also effects our philosophy of life. Hence mathematics plays a vital role in developing our cultural heritage. Young J.W. has remarked that, "whenever we turn in those days of iron, steam, and electricity we find that mathematics has been the pioneer. Were its backbone removed, our material civilization would inevitably collapse. Hence mathematics shapes culture as a playback pioneer and has played an important role in bringing such an advanced stage of development".

D. Artistic or Aesthetic Value

Those who did not get proper opportunities to study mathematics have developed a wrong notion in their minds that mathematics is a dry and unintering subject.

For the lover of mathematics, there is all beauty, art, music and fineness in this subject. One finds a huge treasure of pleasure after getting success in the solution of a mathematics problem. It is the reason why Pythagorus sacrificed hundred oxen to the Goddess for celebrating his discovery of the theorem that goes by his name. In the same way, Archimedes had also forgotten his nakedness after discovering his principle. Just imagine the feeling of ownership and selfconfidence, when one gets success after a long struggle with a mathematics problem. Specially when the answer tallies with the answer given in the book every student feels maximum satisfaction and derives the greatest pleasure.

There is no exaggeration in saying that mathematics is the creator as well as the nourisher and saviour of all the arts. What we enjoy in the arts like drawing, painting, architecture, music or dance etc is all due to mathematics. Mathematics regularity, symmetry, order and arrangement play a leading part in beautifying and organizing the work of these arts. Even the poetry is not enjoyable without mathematics. Music is nothing but the mathematically organized sound. All the musical instruments Harmonium, Drums, table, Flute, Guitar, Sitar, Violen etc are played on the set rules of mathematics. Therefore, Leibnitz is right when he says, "Music is a modern hidden exercise in Arithmetic of a mind unconscious of dealing with numbers".

In dancing, too one has to take care of mathematics in taking steps and responding to the tunes. Moreover, the secret of the beauty of a garden, an ornament or a flowering pot lies in the hands of the arrangement made with the help of mathematics. In brief we can say that secret of the beauty no matter whatever, lies in regularity, precision symmetry, order and arrangement and it is only mathematics that is itself capable of decorating a thing with so many characteristics.

E. Intellectual Value

The study of mathematics helps us to develop all over intellectual powers like power of imagination, memorization, observation, invention, concentration, originality, creativity, logical thinking and systematized reasoning. Every problem in mathematics is an open challenge to the faculties of the mind and a systematic and organized exercise for one's mental health. There is no end of the knowledge in the world of ours. The treasure of the knowledge is not going to be decreased in future but is bound to get more and more enriched. It is rather impossible to acquire such ocean of knowledge in one's life. Therefore the important thing is not to acquire knowledge but to learn how to acquire it. In other words we should aim to acquire the power of acquiring knowledge. Moreover the knowledge is useful only when we know how to use or apply it in solving our problems. The man who has a vast storage of knowledge but cannot use it at the proper time is like a donkey who is loaded heavily without making any use of its load. Professor Schultze says, "Mathematics is primarily taught on account of the mental training it affords and only secondarily on account of the knowledge of facts it imparts," Therefore both the things - the power of acquiring knowledge and skill to use the acquired knowledge properly at the hour of need are aimed through the teaching of mathematics.

Problem solving in mathematics is helpful in the proper development of one's mental powers. Every problem in mathematics passes through a process that

trains an individual in the scientific method of reasoning and thinking. First of all the problem is studied and analysed to know what is given and what is to be found out. Then all the relevant facts and techniques concerning the solution of problem are collected. All of these facts and techniques are carefully analysed and sorted out for choosing the most suitable ones. Now with the help of the chosen technique or sorted facts one tries to reach on some conclusion. The derived conclusions are again verified and accepted only when they prove true in the like-wise situations. Therefore, the problems in mathematics give enough opportunities for the training of the thought process and developing the faculties of the mind. Mathematics emphasizes originality than a mere reproduction and appeals more to the reasoning power than to memory. It increases our power of concentration, reasoning and discrimination and thus helps in developing our mental powers. As Plato observes, "Mathematics is the subject which provides an opportunity for the training of the mind."

F. Disciplinary Value

Mathematics does not only help in developing and controlling the faculties of an individual, it also equips him with proper intellect, reasoning and seriousness needed to lead a responsible life. That is why a mind trained through the study mathematics is more capable of leading a well disciplined life. Study of mathematics is helpful in having constructive discipline. Every student of mathematics is habitual to think properly without any unnecessary biases and prejudices. He can discriminate what is good and what is bad, therefore, he does not take decisions through his emotions but tries to apply the logic and intellect. He does believe hear saying but tries to investigate the thing before reacting to it.

Mathematics by its very nature helps the students to imbibe so many virtues and good habits like concentration, hard working, punctuality, regularly, neatness, cleanliness and orderliness, the habit of paying attention to the classroom study and doing regular home and drill work. These habits go in a long way to train the students in leading a life full of self-restraint and reasoning. That is why a mind trained by mathematics is more disciplined than the mind that is not being trained by mathematics.

G. Vocational Value

The main aim of education is to help the children to earn their living and to make them self dependent. To achieve such aim, mathematics is the most important subject than any other. At present the vocational value of engineering, technology, management, and information technology has become more important and prestigious or reputed. The knowledge and training of these vocations is possible only through mathematics. Almost each and every vocation needs the knowledge of mathematics. Even to learn different vocations related to different branches of science require the knowledge of mathematics.

For example—

1. An architech cannot become a good designer without the knowledge of geometrical drawing and measurement.
2. Official work requires the knowledge of mathematics.
3. To become an engineer, accountant, banker etc, there is need of mathematical knowledge.
4. Similarly to understand different sciences, knowledge of mathematics is essential.

Therefore, it can be said that each and every person needs mathematical knowledge for the earning and to maintain his living standard.

H. International Value

Mathematics has international value in the sense that it is helpful in creating international understanding and brotherhood. Its history reveals that there was a time when our ancestors were not able to count even more than one. Therefore, as far as the potential of knowledge and intellectual development is concerned all human beings- the inhabitants of all the races- are the same and therefore, it is unwise to think superior or inferior to any of the other race, religion, culture or nation.

What we have in mathematics today is the net result of the combined efforts of all the nations and races. Mathematics is not an exclusive property of a particular nation or race. All mathematicians, irrespective of their caste, colour or creed, have generously contributed towards the progress of mathematics. The man made boundaries cannot restrict or check the cooperation among the mathematicians of the world . In fact co-operation and acquaintance with the others' progress is very much essential in going ahead in the field of mathematics and science. This need has brought the nations and races together. The mathematical books and research journals are also exchanged and circulated among almost all the nations of the world. All these things add to the feeling of international understanding and are helpful in bringing international peace.■

CHAPTER - 2

STIMULATING AND MAINTAINING INTEREST IN MATHEMATICS

It may be taken as axiomatic that students will work most diligently and most effectively at tasks in which they are genuinely interested. To create and maintain interest becomes, therefore, one of the most important tasks of the teacher of mathematics. It is also one of the most difficult problems which the teacher encounters. The interest of students is capricious. They are easily caught by any new thing, but easily distracted to other new things. Thus the motivation of the work in mathematics has two aspects, viz, that of creating and arousing interest and that of maintaining the interest. Students readily become interested in things which are new or exciting, in things for which they can perceive practical values or applications to situations and fields of study in which they are already interested, and in things which involve puzzle elements or elements of mystery. The elements of novelty, of usefulness, and of sheer intellectual curiosity are the primary stimuli for the awakening of interest. The work in mathematics

must be organized and conducted so as to emphasize the values and the inherent intellectual challenge of the subject and to ensure understanding and a reasonable degree of competence by keeping the subject matter and the activities at a level of difficulty appropriate to the intellectual maturity of the students.

Motivation through Intellectual curiosity

The average student in the school is an incurably curious individual, and the range of his potential intellectual interests is practically unlimited. Many teachers have never recognized the power of sheer intellectual curiosity as a motive for the highest type of work in mathematics, and as a consequence they have failed to organize and present the work in a manner designed to stimulate the student's interest through a challenge to his curiosity. It could be seen that all theorems of geometry are set up as exercise in establishing certain pre-stated conclusions rather than an exercise for free exploration of the consequences of certain hypothesis. So the element of discovery of the central fact or relationship in each theorem is removed at the outset, whereas this element of discovery could in many cases be retained and used to quicken the interest of the students.

People are interested in seeing how numbers behave and algebra is the science of the behaviour of numbers. Puzzle problems in mathematics have often been criticized as being "unreal" or "having no genuine application to life situations". But with a little experience in teaching algebra it can be convinced that problems do not need to represent, "real" situations in order to be interesting to students. The presence of the puzzle element in problems is of greater stimulus than those elements of so- called "reality" which are usually incorporated in the problem. Therefore, within the frame work of the systematic organization of a course in mathematics there are many opportunities for motivating the work by deliberate stimulation of the curiosity of the students.

Motivation through Application to other Fields of Study

The relation of mathematics to other fields of study often provides an important means of stimulating interest. Remarkable advances have been made through the application of mathematical procedures to advanced studies in genetics, heredity, nutrition, growth and maturation and many other special phases of biological and physiological study. The social sciences also draw heavily upon mathematics, particularly statistical and graphic methods, for the investigation and interpretation of social phenomena. The industrial arts require mathematics. Psychology is finding more uses for it. Even English, the foreign language, and the fine arts are enriched by an understanding of the mathematical principles of form and number, of symmetry and order, upon which they are based. By impressing upon the students these relationships and applications of mathematics to other school subjects, teachers can stimulate interest in the study of mathematics.

Motivation through Application of Mathematics in Professional Fields

A very important means of stimulating interest in mathematics is through pointing out its applications to fields of work through which people gain their livelihood. It should be emphasized that professional work in a number of fields requires intensive training in mathematics and continual use of this subject. The attention

of students should be called to the fact that mathematics is now coming to be recognized as a necessary part of the professional equipment in field like anatomy, physiology, psychiatry etc. In the field of business, executive positions generally require the ability to gather, organize, analyze, and interpret complex statistical data and that the businessman who has been trained in statistical methods has a notable advantage over the one who lacks such training.

Motivation through Emphasis on Cultural and Educational values

While the practical motive for the study of mathematics is a powerful one, teachers should not neglect to point out its cultural and general educational values. It should be the responsibility of the teachers to emphasize continually that it is an essential part of culture and education to understand the background and nature of the developments which are going on in the world. Students should be impressed with the fact that many of these important developments which directly affect our daily lives cannot be adequately understood except through an understanding of scientific principles whose development, expression, and interpretation depend, in turn, upon mathematical principles. The postulation method in mathematics has a major contribution to the cultural education of the individual. An appreciation of the significance of the "if then" type of reasoning is one of the most important potential, educational and cultural values of the study of mathematics. When once attained, it in turn not only makes mathematics infidelity more meaningful but infuses a keener interest into the study and provides one of the most powerful motives for the continued pursuit of work in this field.

Motivation through Mathematics Clubs and Recreations

It is a rare individual, especially child, who is not interested in games or in things which are unusual or unsuspected and which contain elements of surprise or of mystery. While mathematical puzzles, contests, and games cannot be permitted to take too much of the time allotted to regular class work, there is abundant evidence that the moderate and appropriate employment of such devices does add much of interest and zest to the courses.

Mathematics clubs provide an excellent means of stimulating and fostering mathematical study. Such clubs offer excellent opportunities for free consideration of matters of special interest to the members without the necessity of having the programs follow any particular organic sequence of topics such as is generally necessary in regular class instruction. Mathematical recreations are valuable and legitimate in relieving the tedium of necessary routine work and in presenting an aspect of mathematics the existence of which is at times not even suspected.■

CHAPTER - 3

AIMS AND OBJECTIVES OF TEACHING MATHEMATICS

In life, objectives help to focus our attention and efforts; they indicate what has to be accomplished. In education, objectives indicate what the students have to learn; they are explicit formulations of the ways in which students are expected to be changed by the educative process. Objectives are especially important in teaching because teaching is an intentional and reasoned act. Teaching is intentional because it is primarily to facilitate student learning. Teaching is reasoned because what teachers teach their students is judged by them to be worthwhile.

The reasoned aspect of teaching relates to what objectives teachers select for their students. The intentional aspect of teaching concerns how teachers help students to achieve the objectives, that is, the learning environments the teachers create and the activities and experiences they provide. The learning environment, activities, and experience should be aligned with, or be consistent with, the selected objectives.

Need For a Taxonomy

What can teachers do when confronted with what they believe to be an exceedingly large number of vague objectives? To deal with the problem of vagueness, they need to make the objectives more precise. In a nutshell, teacher need an organizing frame work that increases precision and, most important, promotes understanding. A taxonomy is a special kind of frame work. In a taxonomy the categories lie along a continuum. The objectives are classified in a taxonomy. A statement of an objective contains a verb and a noun. The verb generally describes the intended cognitive process. The noun generally describes the knowledge students are expected to acquire or construct. The most useful form for stating objectives is to express them in terms which identify both the kind of behaviour to be developed in the student and the content.

The Structure of Objectives

The domain of objectives is presented as a continuum ranging from quite general to very specific. Along this continuum, Krathwohl and Payne (1971) identified three evels of specificity called global, educational and instructional guidance objectives, with the later commonly referred to as instructional objectives.

Global objectives are complex, multifaceted learning outcomes that require substantial time and instruction to accomplish. A global objective is something presently out of reach; it is something to strive for, to move toward, or to become. For teachers to use global objectives in their planning and teaching, the objectives must be broken down into a more focused, delimited form. Educational objectives describe student behaviour and some content on which the

behaviour will be performed. Educational objectives occupy the middle range on the objective continuum. As such, they are more specific than global objectives but more general than the objectives needed to guide the day-to-day class room instruction that teachers provide. The purpose of instructional objectives is to focus teaching and testing on narrow, day-to-day slices of learning in fairly specific content areas.

Relationship of Global, Educational and Instructional Objectives

LEVEL OF OBJECTIVES			
Scope	Global	Educational	Instructional
Time Needed	Broad One or One or more years	Moderate Weeks or Months	Narrow Hours or Days
Function	Provide vision curriculum	Design Plans	Prepare lesson

In terms of scope, global objectives are broad, whereas instructional objectives are narrow; that is, global objectives do not deal with specifics, and instructional objectives deal only with specifics. Global objectives may require one or even many years to learn, whereas instructional objectives can be mastered in a few days. Global objectives provide vision that quite often becomes the basis for support for educational programmes. At the other end of the spectrum, instructional objectives are useful for planning daily lessons.

Bloom's Taxonomy

Following the 1948 Convention of the American Psychological Association, B.S. Bloom took a lead in formulating a classification of "the goals of the educational process". Three domains of educational activities were identified: Cognitive, Affective and Psychomotor. Cognitive is for mental skills, Affective is for growth in feelings or emotional area, while Psychomotor is for manual or physical skills. Bloom and his co-workers established a hierarchy of educational objectives, which is referred to as Bloom's Taxonomy, where attempts are made to divide objectives into subdivisions ranging from the simplest behaviour to the most complex.

Cognitive domain is demonstrated by behaviour related to intellectual skills : comprehending information, organizing ideas, analyzing and synthesizing data, choosing among alternatives in problem solving, and evaluating ideas or actions.

Affective domain is demonstrated by behaviour indicating attitudes, awareness, interest, attention, concern, ability to listen and respond in interactions with others.

Psychomotor learning in demonstrated by physical skills like co-ordination, dexterity, manipulation, grace, strength, speed and actions which demonstrate the fine motor skills such as use of instruments or tools.

Instructional Objectives: Cognitive Domain

Knowledge

Knowledge is remembering of previously learned material. This may involve the recall of a wide range of materials, from specific facts to complex theories. In the knowledge level, the child knows only the information. This does not mean the information is meaningful to the student. Knowledge represents the lowest level of learning outcomes in the cognitive domain.

Comprehension

Comprehension is defined as the ability to grasp the

meaning of material. This may be shown by translating material from one form to another (words to symbols) by interpreting material (estimating future trends, predicting consequences or effects). These learning outcomes go one step beyond the simple remembering of material, and represent the lowest level of understanding.

Application

Application refers to the ability to use learned material in new and concrete situations. This may include the application of such things as rules, methods, concepts, principles, laws and theories. Learning outcome in this area require a higher level of understanding than those under comprehension.

Analysis

Analysis refers to the ability to break down material into its component parts so that its orginational structure may be understood. This may include the identification of parts, analysis of the relationship between parts, and recognition of the organizational principles involved. Learning outcomes here represent a higher intellectual level than comprehension and application because they require an understanding of both of the content and the structural form of the material.

Synthesis

Synthesis refers to the ability to put parts together to form a new whole. Learning outcomes in this area stress creative behaviours, with major emphasis on the formulation of new patterns or structure. Examples of learning objectives at this level are: write a well organized theme, propose a plan for an experiment, integrate learning from different areas into a plan for solving a problem, formulate a new scheme for classifying objects.

Evaluation

Evaluation is concerned with the ability to judge the value of material for a given purpose. The judgment are to be based on definite criteria. Learning outcomes in this area are highest in the cognitive hierarchy because they contain elements of all the other categories, plus conscious value judgments based on clearly defined criteria. Examples of learning objectives at this level are : judge the logical consistency of written material, judge the adequacy with which conclusions are supported by data.

Instructional Objectives: Affective Domain

Many of the behaviours of an individual are controlled more by feelings and emotions rather than by cognition. A student of mathematics who develops genuine interest in reading books related to mathematics, organizing mathematical recreational activities like puzzle context is a person who has undergone changes in his feelings and emotions. Such behaviours are said to be changes in the affective domain. The instructional objectives are:

Receiving

This means paying attention towards something. It indicates the ability of an individual to receive information. Awareness of the sources of information and willingness to receive the informàtion are the sub levels of this category.

Responding

The learner participates actively in the process. Regularity in attention and motivation leads to responding.

Valuing

Valuing is based on the internalization of a set of specified values. The learner understands that the activity he is called upon to perform is valuable.

Organizing

The learner organizes values into priorities by contrasting different values, resolving conflicts between them, and creating a unique value system.

Characterization

The values are imbibed and the learners behave in a pervasive, consistent, predictable manner.

Instructional Objectives: Psychomotor Domain

Along with the behavioural changes in the cognitive and affective domain the learner has to master certain skills to make the development complete. Speaking, Reading, Writing, Drawing are activities where motor co-ordination is required. Changes of this category leading to mastery of skills are said to be changes in the psychomotor domain. The instructional objectives are:

Perception

The ability to use sensory cues to guide motor activity. This ranges from sensory stimulation, through cue selection, to translation.

Set

Readiness to act. It shows the desire to learn a new process.

Guided Response

The early stage in learning a complex skill that includes imitation and trial and error. Adequacy of performance is achieved by practicing.

Mechanism

This is the intermediate stage in learning a complex skill. Learned responses have become habitual and the movements can be performed with some confidence and proficiency.

Complex Overt Response

The skillful performance that involves complex movement patterns. Proficiency is indicated by a quick, accurate, and highly co-ordinated performance, requiring a minimum of energy. This category includes performing without hesitation, and automatic performance.

Adaptation

The learner responds effectively to unexpected experiences. Skills are well developed and the individual can modify movement patterns to fit special requirements.

COGNITIVE DOMAIN

OBJECTIVES	BEHAVIOURAL/KEY WORDS VERBS
Knowledge	Recalls, Recognizes, Names, Reproduces, Outlines, States.
Understanding	Identifies, Converts, Distinguishes, Estimates
Explains	Generalizes, Translates, Compares, Classifies, Detects errors, Paraphrases, Discriminates
Application	Computes, Manipulates, Solves, Relates, Predicts, Constructs, Selects
Analysis	Analyses, Breakdown, Associates, Examines
Synthesis	Combines, Compiles, Designs, Summarizes, Creates
Evaluation	Appraises, Justifies, Criticizes, Defends.

AFFECTIVE DOMAIN

OBJECTIVES	BEHAVIOURAL/KEY WORDS VERBS
Receiving	Asks, Gives, Points to, Chooses
Responding	Answers, Performs, Recites, Reports, Discusses
Valuing	Initiates, Invites, Joins, Shares, Demonstrates
Organizing	Adheres, Alters, Formulates, Relates
Characterization	Influences, Qualifies, Solves, Verifies, Modifies

PSYCHOMOTOR DOMAIN

OBJECTIVES	BEHAVIOURAL/KEY WORDS VERBS
Perception	Detects, Identifies, Isolates
Set	Begins, Displays, Reacts
Guided Response	Copies, Traces, Reproduces, Responds
Mechanism	Assembles, Constructs, Manipulates
Complex Overt Response	Dismantles, Fastens, Mends
Adaptation	Alters, Rearranges, Adapts.

Behavioural Objectives / Specifications

Behavioural objectives or performance objectives or specifications are terms that refer to a description of observable student behaviour or performance. Since learning cannot be seen directly, teachers must make inferences about learning from evidence they can see and measure. Behavioural objectives provide an ideal vehicle for making inferences. A well constructed behavioural objective describes an intended learning outcome. It communicates the conditions under which the behaviour is performed, a verb which defines the behaviour itself and the degree (criteria) to which a student must perform the behaviour.

In short instructional objectives are not directly observable and measurable. But their evidence of attainment can be collected through specifications.

Taxonomy Reframed: Educational Objectives for the 21st Century

The Taxonomy of Educational Objectives was one of the outstanding research work in the field of education. It is a frame work for classifying statements of what we expect or intend students to learn as a result of instruction. The frame work was conceived as a means of facilitating, the exchange of test items among faculty at various universities in order to create banks of items, each measuring the same educational objective. The revision of this frame work was developed in much the same manner forty five years later by Loren Anderson. (a former student of Benjamin Bloom). Here after, this is referred to as the Revised Taxonomy.

Revised Bloom's Taxonomy: From One Dimension to Two Dimensions

In the original taxonomy, the knowledge category

embodied both noun and verb aspects. The noun or subject matter aspect was specified in knowledge's extensive subcategories. The verb aspect was included in the definition given to knowledge. This brought uni-dimensionality to the frame work at the cost of a knowledge category that was dual in nature and thus different from the other Taxonomic categories. This anomaly was eliminated in the revised taxonomy by allowing these two aspects, the noun and verb, to form separate dimensions, the noun providing the basis for the knowledge dimension and the verb forming the basis for the cognitive process dimension.

The Knowledge Dimension

The one- dimensional form of the original taxonomy becomes a two- dimensional table with the addition of the products of thinking (various forms of knowledge). Forms of knowledge are listed in the revised taxonomy as factual, conceptual, procedural and metacognitive.

Factual knowledge is knowledge that is basic to specific disciplines. This dimension refers to essential facts, terminology, details or elements students must know or be familiar with in order to understand a discipline or solve a problem in it.

Conceptual knowledge is knowledge of classifications, principles, generalizations, theories, models, or structures pertinent to a particular disciplinary area.

Procedural knowledge refers to information or knowledge that helps students to do something specific to a discipline, subject, and area of study. It also refers to methods of inquiry, very specific or finite skills, algorithms, techniques and particular methodologies.

Meta cognitive knowledge is the awareness of one's own cognition and particular cognitive processes. It is strategic or reflective knowledge about how to go about solving problems, cognitive tasks, to include contextual and conditional knowledge and knowledge of self. It is of increasing significance as researchers continue to demonstrate the importance of students being aware of their meta cognitive activity, and then using this knowledge to appropriately adapt the ways in which they think and operate.

The Cognitive Process Dimension

The original number of categories, six, was retained, but with important changes. Three categories were renamed, the order of two was interchanged, and those category names retained were changed to verb form to fit the way they are used in objectives. Like the original taxonomy, the revision is a hierarchy in the sense that the six major categories of the cognitive process dimension are believed to differ in their complexity, with Remember being less complex than Understand, which is less complex than Apply, and so on.

Comparison of Old Version with New Version of Taxonamy

The Taxonomy Table

Old version	New Version
Evaluation	Creating
Synthesis	Evaluating
Analysis	Analyzing
Application	Applying
Comprehension	Understanding
Knowledge	Remembering

Conclusion

The revision's primary focus is on the taxonomy in use. Bloom's taxonomy was traditionally viewed as a tool best applied in the earlier years of schooling (primary and junior primary years). The revised taxonomy is more universal and easily applicable at elementary, secondary and even tertiary levels.

In the national and international scenario, there is a newer reading in the teaching - learning process especially after the wider infusion of the cognitive and constructive psychological interpretations on learning. Gardner's explorations of human intellect in terms of Multiple Intelligence theory triggered this paradigm shift from behaviorist to a new wider perspective. In essence, the wider lacuna that exist in the contemporary teaching- learning process in terms of evaluation can be minimized by incorporating the essential spirit of revised Bloom's Taxonomy.

A list of Objectives and Specifications to be followed in Mathematics classrooms

1. Objectives of the Cognitive domain.

I. The learner gathers information (acquires knowledge) of mathematical terms, symbols, concepts, etc.

Specifications: The learner:

i. recalls the terms, symbols etc.

ii. recognizes the terms, symbols etc.

II. The learners comprehends (or develops understanding of) the terms, symbols, concepts, etc.

Specifications: The learner:

i. closely observes the phenomena.

ii. compares items on the basis of the attributes.

iii identifies relations.

iv. identifies mistakes committed in the process of comprehending.

v. classifies objects or phenomena in terms of the attributes.

vi. defines concepts in terms of the attributes.

vii. gives ones own illustrations or examples for a concept.

III. The learner applies the learned information in new or unfamiliar situations.

Specifications: The learner:

i. identifies the purpose for application.

ii. makes the unfamiliar familiar.

iii. identifies the relation among the data available.

iv. hypothesis a plan of action for solution.

v. tests the adequacy of the data for facing the new situation.

vi. finds out additional data, if required.

vii. arrives at generalizations.

viii. judges whether the conclusions arrived at are valid.

ix. establishes new relations or conclusions.

x. makes correct estimation.

xi. verifies every step on the basis of principles.

xii. synthesizes the process involved in the solutions and summarizes the results systematically.

xiii. at every stage monitors the progress along the correct path.

2. Objectives of the Affective domain

I. The learner develops healthy interest related to mathematics.

Specification: The learner:

i. attends to articles related to mathematics from various media.

ii. reads books on great mathematicians.

iii. regularly uses mathematics library and mathematics laboratory.

iv. likes company of mathematically talented persons.

v. derives pleasure in solving mathematical puzzles.

vi. creates puzzles on ones own.

vii. seeks necessary clarifications from the teacher.

viii. writes articles on the subject in the magazine.

ix. collects illustrations, aids related to mathematics.

x. prepares albums to preserve collections related to the subject.

II. The learner develops scientific attitude towards mathematics

Specifications: The learner:

i. refrains from jumping into conclusions.

ii. accepts a proposition when and only when it is logically proved.

iii. accepts errors without hesitation.

iv. points out errors in facts, arguments, etc.

v. exhibits habits of questioning.

vi. shows eagerness to probe into problematic situations.

vii. maintains precision, clarity, logical stability in verbal or written statements.

III. The learner appreciates the beauty of mathematics.

Specfications: The learner:

i. appreciates the rhythm and beauty of geometrical and number patterns.

ii. gets pleasure while experiencing the accurate and precise way of mathematical arguments.

iii. constructs beautiful pattern using geometrical shapes.

3. Objectives of Psychomotor domain

I. The learner develops skill in

a. computation

Specifications: The learner:

i. performs the four primary operations with speed and accuracy.

ii. uses short cut methods to make calculations.

iii. works out computation mentally.

iv. records the written form of computations systematically and clearly.

b. drawing geometrical figures and graphs

Specifications: The learner:

i. takes measurements accurately using instruments.

ii. estimates length, area etc without using the instrument.

iii. selects appropriate scales for drawing plans.

iv. does free hand drawing fairly.

v. draws geometrical figures and graphs accurately using appropriate instruments.

vi. labels figures or graphs correctly.

c. interpreting given figures

Specifications: The learner:

i. interprets graphs and charts correctly.

ii. identifies the relation connecting the various parts of a figure, graph or tables.

iii. converts the measures in a diagrammatic representation to real measures with accuracy.■

CHAPTER - 4

DIFFERENT METHODS OF TEACHING

Teaching Method: Meaning

A teaching method is a course of action meant for realizing a preconceived educational goal through a series of teacher - pupil activities appropriate for the purpose. The method of teaching refers to the regular ways or orderly procedure employed by the teacher in guiding the pupils, in order to accomplish the objectives of learning situations. According to Brandy (1963), "Method refers to the formal structure of the sequence of acts commonly denoted by instruction. The term method covers both strategies and techniques of teaching and involves the choice of what is to be taught".

While selecting the method, the teacher should always keep in mind the aims of teaching mathematics. These aims include the mental, social and moral development of the child. But a method should not become an end in itself. The teacher is free to use a variety of

teaching methods. Following points may be considered while selecting a method :

- What to teach?
- Why to teach?
- Whom to teach?
- How to teach? What are the various methods?
- What are the problems in using these methods?
- How can the problems be rectified?
- Which method is the best?

Characteristics of Good Teaching Methods

Following are the major characteristics of a good teaching method:

1. It should provide a variety of related experiences and activities meant for individuals as well as groups and should be designed to bring about anticipated developmental changes in pupils.
2. It should give scope for the creative expression of the child's individuality.
3. It should be capable to promote interest among the students
4. It should train the students in the technique of self -study and the method of processing informations though personal effort
5. It should stimulate the desire for further study and exploration.

Classification of Methods

Generally, methods of teaching mathematics can be classified in two categories.

- Child - centred methods
- Teacher - centred methods

Child - centred methods

In child - centred methods, the child occupies a central position in the classroom. The whole teaching - learning process is geared to the needs, interests, capabilities and requirements of the child. The methods in this category include: Project, Laboratory, Problem-solving, Heuristc, Inductive-Deductive, Analytic-Synthetic etc.

Teacher - centred methods.

In teacher - centred methods, the teacher occupies a central position in the classroom. Focus is given on telling, memorization and recalling information. Methods in this category include : Lecture, Historical and Lecture - cum - demonstration method.

Various Methods of Teaching Mathematics

Various methods of teaching mathematics are as follows :

1. Lecture Method
2. Demonstration Method.
3. Lecture cum Demonstration Method
4. Inductive - Deductive Method
5. Analytic - Synthetic Method
6. Laboratory Method
7. Heuristic Method
8. Project Method
9. Problem - solving Method

Lecture Method

This is teacher - centred method. Teacher is an active participant and the child is a passive learner. This is not a psychological method. It is a method of imparting information though speech. Lecture method is useful at higher level classes. It can also be used to relate some of the historical and mathematical incidents. But it is difficult to know the extent to which the student has been able to learn.

When to use Lecture Method

- To introduce the new lesson and new topics
- To develop interest and to motivate the child
- To correlate new knowledge with the previous knowledge.
- To summarize the lesson or content which has been taught.
- To fulfill queries and information.
- When a large number of children are to be taught at the same time.

Steps in Lecture Method

There are three steps in the process of lecture method:

1. Planning by the teacher
2. Presentation by the teacher
3. Receiving by the learner

Merits of Lecture Method

- This is an easy, attractive and brief method
- It is useful to give fact based knowledge and historical development of mathematics.
- A single teacher can teach a large group of students at a time.
- It is more useful for higher classes.
- It is useful for introducing new knowledge.

Demerits of Lecture Method

- This method is against psychological principles.

- The learner remains passive and inactive.
- It does not help in developing mental abilities like reasoning, logical thinking etc.
- There is no provision for practical and creative work.
- The knowledge imparted though lecture is not durable.

If a lecture is properly planned and prepared, it may help in inspiring, stimulating and motivating learners. One of the procedures for preparing a lecture may be as follows.

1. Aims and Objectives of Lecture.

   (Why and for what it is planned)
2. Introduction of the lecture / topic.

   (Relating with previous knowledge of the learners)
3. Organisation of the content.

   (Major points, Development of Points, Questions, examples)
4. Presentation of the lecture.

   (Discussion, Demonstration, Effective voice, use of gestures)
5. Conclusions and Summary.

   (Asking questions to students)

Demonstration Method

In mathematics, teaching by demonstration method is very important. The teacher makes a theoretical investigation and proves it in the classroom. It involves presentation of a pre-arranged series of events to a group for their observations.

While demonstrating teacher should ask some reflective type of questions to stimulate the power of reasoning and interest of students in the classroom. This is commonly used in science and fine arts.

Merits

- This method shortens the time for learning and lengthens the memory of facts. It fosters good thinking in pupils.
- It brings about a relationship between theory and practice.

Demerits

- First hand experience is not possible with this method.
- Cost is high.
- It is a time consuming method.

Criteria of a Good Demonstration

- The purpose of demonstration must be stated to the student.
- Experiments chosen should be simple and clear.
- Demonstration table should be ready with equipments in working condition and must be arranged in order.
- Involvement of the students in the process is essential.
- Teacher should discuss with the students the observation and results of the demonstration.

Lecture - Cum - Demonstration Method

This is a modified form of lecture method. Demonstrations serve a useful purpose in teaching. The students' keen observation during demonstration enables them to comprehend ideas meaningfully through the related 'enaction'. Demonstration involves observation of a skill, a process, a phenomenon functioning of a systems or a working model. The effect of observation is enhanced by the explanation provided. This strategy of using both demonstration and lectures at the same time is known as the lecture - cum - demonstration method.

Inductive Method

Inductive Method of teaching and learning mathematics is based on induction. Induction means proving a universal truth by showing that if it is true in any particular case, it will be true in the next case and it will be true for all such cases. The learner is made to discover truth by himself. Rules and formulae are established after extensive study of experiments and examples. Inductive method is more useful in lessons where principles, rules, definitions, generalizations and casual connections between facts are to be established.

Example - Binomial expansion $(a + b)^n$ is named as method of induction.

In this method learners are helped in discovering a formula by adopting inductive reasoning. In inductive reasoning one proceeds from particular to general, from concrete facts to abstract rules and from specific examples to the general formula. If one rules applies to a particular case and is equally applicable to different similar cases, it is accepted as a generalized rule or formula.

Steps in Inductive Method

While teaching by this method the following steps are mainly used

i. Presentation of specific examples

Teacher presents many examples of same type and solutions are obtained with the help of learners.

ii. Observation

After obtaining the solutions the learners observe these and try to reach some conclusion with the help of teacher.

iii. Generalization

After observing the examples, the learners decide some common formulae, principle or law.

iv. Testing and verification

After deciding, the common formula or principle, learners test and verify the principle with the help of other examples.

Merits

- ♦ It is a scientific method because knowledge attained is based on real facts.
- ♦ The critical observation and logical power of learners are developed by this method.
- ♦ Many principles of psychology are used in this method.
- ♦ It develops self-reliance and self confidence.
- ♦ This method is very useful for lower classes.
- ♦ It develops the curiosity and interest of the learner in the subject
- ♦ It discourages cramming.
- ♦ This method is based on self - activity.
- ♦ It provides opportunity for good teacher - pupil relationship.
- ♦ It is a natural method of making discoveries.

Demerits

- ♦ It is time consuming and laborious method.
- ♦ Learners of higher classes may not be able to hold their interest in this method.

- All topics in mathematics cannot be covered by this method.
- It is not always possible to present real examples for generalizations.
- Problem solving skill in the learners cannot be developed by this method.

Deductive Method

Deductive Method is the inverse of inductive method. It is based on the principles of going from abstract to concrete, general to particular and formula to problem. Here a pre-established formula is given to the learners and they are asked to use the same in solving a number of relevant problems. Deductive reasoning is used in this method. Deductive reasoning begins with generalizations or deducted results. It is mainly used in Algebra, Geometry and Trigonometry.

Merits

- This method is useful for revision and drill work.
- It develops the memory power of the learner.
- It is a time saving method. Learner's speed of work can be developed. Laws and principles can easily be checked by this method.
- It completes the process of induction

Demerits

- This method is not in accordance with the psychological principles.
- Here more emphasis is given on cramming than understanding or discovery.
- It taxes learner's mind.
- The learners get disinterested as they are only passive listeners.

- There is no scope for developing logical thinking of the learner.
- It is difficult for a beginner to understand an abstract formula.
- Learners are not provided the opportunity to gain new knowledge.

Applications of Inductive-Deductive (Indo-Deductive) reasoning in teaching mathematics

In olden days inductive method and deductive method were considered as two separate instructional strategies. But they are complementary to each other.

Induction is the process of arriving at a generalization by examining a large number of representative individual cases. This will not only make the principle convincing to the learner, but also familiarize him with the logic involved. He is helped to become a discoverer of mathematical truth. Thus inductive reasoning helps the child to maintain the spirit of heurism leading to construction of new knowledge.

Though induction is helpful in meaningfully and independently arriving at generalizations by learners, the subject becomes a logically well-knit 'system' only if deductive reasoning is adopted to interpret new cases. Deduction is the process of applying approved generalizations to comprehend or interpret a particular instance that falls within the domain of the approved generalization, to draw conclusions about the instance considered.

In short, the 'inductive deductive approach' should be practiced for building up a logically sound system in mathematics.

Analytic Method

Analysis is the process of breaking things into smaller parts. Analyzing a problem means breaking it into simpler elements or unfold its hidden aspects in such a way that its solution appears quite obvious. Analysis starts with "what have to found out" and traces the connection between it and the data. The procedure adopted is to go from 'unknown' to 'known' or from 'results to hypotheses'.

The great psychologist Thorndike supported this method. He believed that through analysis, the highest intellectual performance of the mind is possible. This method is mostly used in the following conditions.

- When a theorem has to be proved
- When construction work has to be done in geometry.

Merits

- It leaves no doubts in the minds of the learner as every step is justified.
- It is based on psychological principles.
- This method leads the learner to the spirit of enquiry and investigation.
- Learner's active role in the process results in longer retention.
- Analysis is the process of thinking.

Demerits

- It is a formative method based on inductive reasoning.
- It develops self confidence and reasoning power.
- It is lengthy, time consuming and therefore not economical.
- This method does not help in developing speed and efficiency.

- The syllabus cannot be completed within the time limit.
- This method is not suitable for all topics in mathematics.
- In this method information is not presented in a well organized manner.

Analytic method is very effective for teaching how to solve complex mathematical problems, and in proving theorems and riders. It is particularly useful for solving problems in arithmetic, algebra, geometry and trigonometry.

Synthetic method

Synthetic is derived from the word 'Synthesis'. Synthesis is the complement of analysis. To synthesize is to combine the constituent elements to produce something new. In this method, starting is from something already known and connecting it with the unknown part of the statement. So the procedure is from known to unknown. It is the process of combining known bits of information to reach the point where unknown information becomes obvious and true. Thus analysis is the process of discovering the solution and synthesis is the method of setting out the solution in the concise form.

Merits

- It facilitates speed and efficiency of the learner.
- In this method the discovered facts are presented concisely.
- It glorifies memory.

Demerits

- It leaves many doubts in the minds of the learners.
- As it does not justify all the steps, recalling may not be possible.

- There is no scope for discovery.
- It does not provide full understanding.
- This method makes learners passive and encourages rote memorization.
- It does not provide opportunity for developing thinking power.

Synthetic method is best suited for the final presentation of proofs and solutions to problems in a logical and systematic manner.

Applicability of Analytic- Synthetic Methods

Analysis is often identified with induction and synthesize with deduction. Thorndike remarked "The mind's most intellectual act is to connect one thing with the other, but its highest performance is to think of a thing in terms of its constituent elements". Synthesis is the complement of analysis. In logic and mathematics the two always go together. Moreover analysis leads to synthesis and synthesis makes clear and complete the purpose of analysis. Analysis is the method of discovery and synthesis is the method of elegant presentation. In short, analysis is the instrument used, and the final result is the outcome of the process of synthesizing the analyzed facts.

Laboratory Method

Laboratory method is a procedure for stimulating the activities of the learners and to encourage them to make discoveries. In this method learners are required to do some experiments or carry out certain activities. It is the experimental portion of the inductive method or the practical form of the heuristic method. It is based on psychological principles of learning such as 'learning by doing', 'learning by observation' and from concrete to abstract. Laboratory method is

quite competent to relate the theoretical knowledge with the practical base. This approach makes the learning process more interesting, lively and meaningful.

The success of laboratory method depends on an able and skilled mathematics teacher as well as the availability of a well equipped mathematics laboratory. A well furnished mathematics laboratory helps in providing stimulating and worth while experiences in clarifying the meaning of mathematical principles and for the acquisition of skills.

For the success of laboratory method following points should be kept in mind.

- The different branches like Arithmetic, Algebra, Geometry, and Trigonometry which can be taught with the help of this method should be decided in advance.
- Equipments and other necessary things should be made available.
- Teachers should observe the work of the learners.
- Theoretical knowledge related to the topic should be given before experimenting.
- The steps and procedure should be made clear so that the learners do not face any difficulty while experimenting.
- For the success of this method only small groups should be formed.

Merits

- This method is based on psychological principles.
- The knowledge acquired by this method is more solid and durable.
- It helps in developing the habit of discovery and self-study.

- It helps in the development of observation and logical power amongst the students.
- This method provides an opportunity to verify the validity of mathematical rules through their application.
- It provides opportunity for social interaction and co-operation among the students.
- It develops scientific attitude in the learner.

Demerits

- It is an expensive method.
- This method can be used for a small class only.
- It is time consuming method. The syllabus cannot be completed on time.
- This method does not fit the learners of lower classes.
- It is not easy to prove all mathematical principles, laws and formulae in the laboratory.

This method does not contribute much towards the development of reflective thinking, reasoning and problem solving skills which are important aims of teaching mathematics. Whenever a teacher selects the laboratory method, it should be integrated with other methods to yield desirable outcomes.

Heuristic Method (Heuristic Approach)

The term 'heurism' means spirit of discovery. This approach has been first advocated and popularized by the German Psychologist Amstrong. This is often interpreted as a method of instruction but it would be more justifiable to consider it as an approach that would make the entire instructional methods effective, learner - centred, discovery - oriented and logic - based.

For maintaining this approach, the teacher has to arouse in the minds of the learners a desire to discover by themselves. He should expose the minimum possible, but arouse their curiosity and make them engage in inquiry into problematic situation being considered. They should be made to act on their own and gradually proceed to the destination - say discovery of a principle or finding out the solution of a problem or for designing a process for the production of some new pattern. The teacher may ask heuristic questions that would make the learner probe into the problematic situation concerned. In general, all the strategies applied for discovery will become familiar to the learner by intelligently responding to the heuristic questions posed by the teacher. When once the learner gets familiarized with strategy of investigation adopted by the teacher, he should be given the opportunity to ask such questions himself.

Merits

- This is a psychological method.
- It develops self-confidence, self reliance and scientific attitude.
- It develops ability of observation and spirit of enquiry to solve the problems.
- The knowledge obtained by this method is more stable.
- It provides individual differences as each student can work at his own pace.
- It helps in the development of social skills as the students have to co-operate with one another.
- A good teacher - pupil relationship can be established.

Demerits

- This method is not suitable for lower classes.
- It is a lengthy method.
- There is lack of text books written on heuristic approach.
- It presupposes a very small class, which is not possible in Indian conditions.
- Evaluation can be difficult.

The nature of this approach has the most relevant significance in mathematics education. This is because, it is a subject of study which deals with logically sequenced concepts, principles and processes and their application in new situations related to all areas of study. Moreover, the application of mathematical knowledge is in most cases related to problem solving, which warrants not only knowledge but also a strong spirit of discovery essentially required for analyzing the problems, with a view to reach the solution on ones own. So heuristic approach is most relevant in mathematics education in view of the high degree of logic involved in that discipline.

Project method

This is a method of instruction that is designed in tune with the pragmatic philosophy as applied in teaching and learning. Use of realistic situations and facilities for finding out workable solutions to social problems is the main feature of pragmatic philosophy. The most ardent preacher of this philosophy of thought was John Dewey and the most famous educator who applied its spirit in education was Kilpatric. A project is often defined as 'a problematic act carried to completion in its natural settings'. The project method takes children out of the classroom atmosphere to the realities of actual life – from academic to practical. 'Learning by doing' and 'learning by living' are the two cardinal principles of this method.

Principles of the project method

Some of the basic principles of project method are

i. Purpose:

The project should be purposeful. There should be some set aims for each project and the students should have a clear idea of what they do and why.

ii. Activity:

The project should cater to the natural tendency of the learners. The teacher should allow them to think and plan independently, to exercise their judgment and to workout the project to the best of their ability.

iii. Utility:

The experience gained from the project should be useful. Activities undertaken must be completed and the knowledge gained there must lead to further acquisition of knowledge.

iv. Freedom:

There should be full freedom of the students to work on their own accord. Project should grow out of the learner's own purpose and needs.

v. Economical:

The project should be economical and the purpose of the project should be achieved without any waste of time and money.

vi. Challenging:

The project should be challenging. Psychologists have proved that, learners would prefer to do a task which requires reasonable amount of effort.

vii. Feasibility:

Before giving final approval of the project, its feasibility should be considered.

Steps in project method

An educational project is a whole hearted purposeful activity aimed at the learners. The various stages in the project method and the teacher's function at each of these stages are as follows.

i. Providing a situation

The first step is provision of a suitable situation where the students find some scope for carrying out a useful activity. The teacher should always be on the look out for curricular or co curricular situation that would provide 'problematic act' which could be carried to completion in a natural setting. The situation may be presented in different ways such as, conversing with the class on different topics of interest to the students, taking them for a significant study trip etc.

ii. Choosing and purposing

After the students have been provided with problematic situations, they may be encouraged to choose one among them. The teacher should guide the students in such a way that they get the necessary insight to make a good choice.

iii. Planning

Planning should also be done by the students themselves under the guidance of the teacher. The teacher should initiate by drawing the attention of the students to the need for a plan. In order to realize the output anticipated by the project, a number of inputs will be required. Some of these may be readily available; others have to be procured. Availability of all these has to be ensured before finalising and starting the project.

iv. Executing the project

Every student should contribute actively in the execution of the project. While distributing the work, interest of the students must be considered. The teacher should guide and observe the progress of the project.

v. Evaluation of the project

After the project has been executed, the students must review their work and try to see what mistakes they have committed in planning or in its execution. Whether the work has been carried out in accordance with the plan laid down has to be checked. The quality of output is also very important. The teacher's role is to provide the students with necessary standards of evaluation.

vi. Recording

The students should maintain a complete record of the project work. While recording, some points like – how the project was planned, what discussions were made, how duties were assigned, how it was evaluated etc. should be kept in mind.

Role of Teacher

The role played by the teacher in project method is vital. The following are expected from the teacher.

i. The teacher should provide occasion for every student to contribute something towards the success of the project.

ii. A democratic atmosphere must be created so that the students can express themselves freely.

iii. The teacher should be alert to see that the project is running on its right track.

iv. The teacher should have thorough knowledge and experiences.

v. The teacher should thoroughly study the abilities, interests and aptitudes of the students so that they may be allotted suitable work.

Merits

- It provides a good deal of independence to the pupils.
- It has practical value.
- This method follows psychological principles.
- The different subjects of study can be meaningfully correlated and integrated.
- It emphasize sociability and dignity of labour.

Demerits

- It requires a lot of money.
- Many topics cannot be taught by this method.
- There is no scope for practice.
- It is difficult to choose projects that are socially and educationally sound at the same time.
- Evaluation of the achievement of the learners is a difficult task.
- For successful application of the project efficient and resourceful teachers are needed.
- It upsets the routine work of the school.

Problem Solving Method

Problem solving is the scientific process of solving problems. A problem is a challenge that warrants additional effort on the part of the learner to arrive at feasible solution which in turn will help in realizing many of the deeper level educational objectives. Problem solving is a method in which learner uses his ability to analyse a problem which confronts him in order to arrive at a solution. It involves the use of reflective thinking or reasoning.

A student while studying mathematics may be asked some questions or is confronted with a mathematical problem given in the text book, the answer or solution of which is unknown to him. The process of finding a solution to the problem by reflective thinking may be termed as problem solving. So problem solving is "what one does when he does not know what needs to be done". According to Yokam and Simpson "A problem occurs in a situation in which a felt difficulty to act is realized".

Reflective thinking

Reflective thinking is not a sudden impulsive thought. A few essentials of reflective thinking are:

- Sensing the presence of a perplexing problem.
- Recognizing clearly the nature of the problem.
- Ability to hold the problem in mind as it is studied and not to lose enthusiasm.
- Readiness to make a bold guess as hypothesis by way of solution.
- Ability to examine and evaluate critically the proposed solution.
- Boldness to cast aside hypothesis which has not been found valid.
- Maintaining an attitude of suspended judgement until all facts are gathered, weighed and evaluated.
- Readiness to re-check conclusion and to test their validity.

Characteristics of a problem in a learning situation

A problem should possess the following characteristics if it has to be educationally valuable.

A problem should be:

- clear and definite.
- challenging.
- suitable to the age, needs and capability of pupils.
- related to actual life situations.
- thought provoking.
- correlated to the existing knowledge of the learners.
- worthwhile and of practical value.
- workable with resources available.
- feasible within the time available.

Teacher's role in problem solving

The teacher should:

- maintain the spirit of discovery among students.
- give proper guidance to the students from the beginning till the solution is reached.
- establish rapport with students, for the smooth completion of the work.

Steps in the problem solving method

i. Sensing the problem

The teacher presents a challenging situation in which the students feel the presence of the problem and need for solving it.

ii. Interpreting, defining and delimiting the problem

The teacher helps the pupils through heuristic questions to interpret and identify the exact problem involved.

iii. Collecting the relevant data

Encouraging pupils to hypothesize probable paths leading to the solution will make data gathering objective - based.

iv. Organising and evaluating the data

The data collected are then properly organized and evaluated. The unnecessary or irrelevant data are avoided.

v. Formulating tentative solution

The pupil formulates hypotheses on feasible solutions.

vi. Arriving at the final solution

The tentative hypotheses are pooled together and tested for acceptance or rejection. Discussion and argument by the pupils with intervention of the teacher are necessary.

Problem solving method is an appropriate method for teaching mathematics. There are various approaches to problem solving namely analytic, synthetic, inductive and deductive approaches. Problem - solving method makes use of any of these or a combination of these approaches.■

CHAPTER - 5

PLANNING FOR TEACHING

Necessity for Planning

The success of any work depends on how thoroughly and systematically it is planned in advance. It is said that "If you fail to plan, you plan to fail and nobody plans to fail but fail to plan". As planning is the key to success in any activity, success of teaching - learning process also depends upon proper and meticulous planning.

Types of planning

Planning for instruction has to be done at various levels and for a variety of purposes. Depending upon the scope and aim envisaged the nature of planning also will vary. Types of planning to be made by teachers are:

Year planning

Year planning is a long term planning of the instructional process. A teacher who teaches a course in mathematics for a particular class at the beginning of

the academic year, plans the curricular and co-curricular activities as per the syllabus for the entire academic year. Such a planning would provide the teachers with a design of the work to be executed during the year as a whole. Year plan in a subject should indicate the course purpose and objectives, course units, number of lessons, the time schedule for dealing with each unit, general suggestions regarding methods of teaching, details of equipment and aids, their sources etc.

The year plan forces the teacher to consider the time available and make the optimum use of it, by planning per term, per month and per week before the commencement of an academic session. The year planning helps the teacher in taking decisions regarding when to teach what, how much time can be assigned for each topic, how many hours of project or laboratory work can be assigned to the students, how much time can be allotted for revision, tests etc. Thus a year plan can regulate instruction through out the year.

Advantages of Year Planning

- It indicates the total weightage to be given for various instructional objectives and content.
- It points out the way of achieving these objectives and the methods and approaches to be adopted for each topic
- It makes evaluation objective based.
- It keeps the teacher on the right track.
- It promotes professional co-operation and mutual exchange of ideas.

Unit Planning

Unit planning is a comprehensive planning for the instruction of a 'unit of a study'. A unit is a unique segment of subject matter in which the various items of knowledge are meaningfully linked to each other as components of a single compact entity. A unit is in fact a 'compound' of lessons and not a 'mixture' of lessons. A unit is a large segment of subject matter having a common theme or idea. A unit can be split up into smaller sub-units called topics and topics are linked to one another by a common idea or a principle.

A unit plan is a design for transacting the curriculum material involved in a 'unit' with predetermined objectives to be realized.

It also suggests the learning activities leading to experiences that would be evidences for the realization of these objectives.

Advantages of a Unit plan

- Unit plan breaks up a lengthy unit into smaller sub-units so that its scope can be grasped quickly.
- The teacher can present the principles and concepts constituting the unit in a systematic way.
- It helps the pupils to see the relationship between the various facts, principles and processes that constitute the unit.
- It enables the pupils to identify the unifying principle linking all the topics.
- It helps the teacher to plan definite outcomes of learning.

Lesson Planning

Lesson plan is a plan of action. It is an experience of anticipated teaching. It includes the working philosophy of the teacher, his understanding of his pupils, his comprehension about the objectives of education, his knowledge of materials to be taught and his ability to utilize effective methods for the realization of pre-determined goals.

John Fredrik Herbart a German philosopher and educationalist is credited with the formation of a systematic lesson plan in education. It consists of content, specification, learning activities and evaluation techniques. To visualize the interrelationship these are written in four columns.

Following are the steps included in a lesson plan.

Introduction

The success of a lesson depends on how well the students are motivated at the beginning of the lesson. A lesson may be introduced effectively by presenting a demonstration, using teaching aids, asking thought provoking questions, relating a story in brief or by testing the previous knowledge. This stage is mainly intended to prepare the minds of the learner to receive new knowledge.

Development

This is the most important step in a lesson. The pupils are made to be active participants through skillful questions. The teacher chooses appropriate learning activities inorder to develop the new facts in a logical order. This stage consumes about two-third of the duration of the period.

Application

At this stage pupils are given the chance to apply the principles, facts, rules etc. It aims at providing opportunities for drill exercises. Common mistakes or the difficulties experienced by the pupils can be noted and this will form the basis for remedial teaching.

Review or Recapitulation

Review helps in recalling the lesson learned and to find out the extent to which teaching is effective. Recapitulation may be done through oral questioning or summarizing the content covered. Oral questions should not involve too much of computation.

Home Assignment

Classroom work alone is not sufficient to provide the much needed practice. Hence home assignments are set in mathematics. It may be given in the form of a few sample problems, preparing a model, drawing a chart, collection of related articles, maintaining an album etc.

Characteristics of an effective lesson plan

A good lesson plan should:

- include clearly mentioned instructional objectives.
- include teaching aids: how and when to be used.
- include motivation and evaluation techniques.
- include student centred activities.
- show connection between previous lessons and future lessons.
- provide sufficient scope for mixed ability groups.
- anticipate pupils' difficulties and questions.
- provide effective illustration through out the period.
- include relevant home assignment.
- provide proper time allocation.
- be relevant to actual situation with reference to the syllabus, timetable and socio-economic cultural background of the learner.

Advantages of a lesson plan

- It helps to think about meaningful learning activities to be provided in the classroom.
- Lesson plan provides an outline of the content.
- It gives the teacher a greater confidence in teaching.
- It establishes proper connection between different lessons and thereby ensures continuity.
- It helps the teacher to anticipate potential difficulties and problems.

Different approaches to lesson planning

1. Herbartian Approach
2. Morrison's or Unit Approach
3. Bloom's or Evaluation Approach
4. RCEM Approach

1. *Herbartian Approach to Lesson Planning*

John Fredrik Herbart, a German philosopher was the first to develop a systematic lesson plan. The Herbartian lesson planning involves Appreciative Mass Theory of Learning. So Herbartian approach gives much emphasis to teacher presentation and also influenced by Classical Human Organisation theory. There are five steps – preparation, presentation, comparison and abstraction, generalization, application.

Outline of the lesson plan

(i) Subject, topic, class with section, period and date
(ii) General objectives of the teaching subject
(iii) Specific objectives related to the topic
(iv) Introduction
(v) Statement of aims

(vi) Presentation including developing questions
(vii) Explanation and Black Board summary
(viii) Review Questions
(ix) Home assignment

2. *Morrison's or Unit Approach to Lesson Planning*

Prof. Morrison was the founder of Unit approach. Unit approach lays emphasis on the unit method for the planning of teaching - learning activities. The teaching learning process must result into the mastery over the subject matter or the contents prescribed for a class. The subject matter is split into meaningful small portions known as units. It involves the following steps.

(i) Exploration
(ii) Presentation
(iii) Assimilation
(iv) Organisation
(v) Recitation.

Steps involved in Morrison's approach and Herbartian approach are similar in nature. First step is preparatory approach with which a pupil teacher explores various possibilities to make his lesson a success. Different methods are devised to motivate the students and to maintain the interest of the students during the whole span of class period.

Second step involves different methods to present the subject matter.

Third stage involves intensive learning, deep understanding of the subject. Students are provided with guide sheet to remove the debts and short comings.

Organization helps in determining the extent to

which the students are able to reproduce the material of the unit in writing without any outside help like key-notes. The ability of the teacher to enable his students to reproduce the lesson taught reflects the efficiency of a teacher. Morrison has described this efficiency of a teacher as a process of organization.

Recitation in unit lesson planning means that an individual student is able to reproduce the same text orally on the completion of the lesson by a teacher. Morrison's recitation also encourages the pupil to explain the unit lesson taught with the help of black-board summary or by laboratory apparatus used by the teacher during the lesson.

3. ***Bloom's Evaluation based approach to lesson planning***

According to Bloom, teaching learning process depends upon three dimensions of our behaviour – cognitive, conative and effective. Bloom's approach involves the following steps.

1. Teaching points or content

In the first column of the lesson planning, the subject matter or content is written in the form of main teaching points.

2. Objectives and their specifications

It is concerned with the writing of educational objectives in clear and concise behavioural terms.

3. Teacher's activities

In this column of the lesson plan, all those activities are mentioned which a teacher performs for the realization of the desired objectives.

4. Student's activities

The activities undertaken by the students for the realization of teaching objectives are mentioned in this column.

5. Teaching aids

Various types of teaching aids used for providing desirable teaching-learning experiences are mentioned here.

6. Evaluation

Evaluation as a step in the lesson plan is an important device to find out the extent to which stipulated objectives have been realized through the teaching-learning act. The result of such evaluation provides needed feed back to both the students as well as to the teacher for bringing desirable improvement in the process of teaching and learning.

4. RCEM *(Regional College of Education, Mysore)* approach to lesson planning

The rationale behind the RCEM approach is a system approach in education. A system approach believes in presenting information in a systematic order. For this purpose, content of the lesson and the objectives are to be defined in behavioural terms. In RCEM approach, three major behavioural objectives are input, process and output.

These steps are also known as Expected Behaviour Outcomes (EBOS),Communication Strategy and Real Learning outcomes (RLOS). These aspects resemble the introduction, presentation and evaluation phases in a usual lesson plan.

The inputs help in identifying the behavioural objectives to be attained.

Process is the most important as this stage involves presentation, integration of knowledge and skills. This stage represents the learning experiences, teaching strategy and tactics, audio-visuals aids, techniques of motivation, ways of securing proper classroom interaction etc.

The input is the entering qualities of an individual and the output indicates the change in behaviour at the time of completion of a course.

RCEM approach to lesson planning depends upon the expected objectives of teaching ie, Expected Behaviour Outcomes (EBOS) being processed by the students or teacher's activities to get Real learning outcomes (RLOS) among the students.

Types of Lesson Plan

Lesson plan is an outline programme of action of a teacher to teach a particular topic in a subject.

There are four types of lesson plan.

1. Development Lesson Plan
2. Practice or Skill Lesson Plan
3. Review Lesson Plan
4. Appreciation Lesson Plan

Development Lesson Plan – DLP

It is also known as the knowledge lesson plan.,DLP helps to develop new learning by making use of the previous knowledge of the children. Any new knowledge should develop out of the previous knowledge and experiences of the child. In preparing a development lesson, emphasis should be given to the way in which how a new material can be linked to the old, and how interest in the new material can be aroused. Common techniques employed in development lessons are narration, questioning, explanation etc.

Skill Lesson Plan

Practice lesson or skill lesson involves teaching skills to develop teaching on a skillful behaviour. To practice skill lesson a pupil teacher should consider

(i) The meaning of the skill to be acquired.

(ii) Skill to be learned should be taught as a whole or in parts.

(iii) Drill or practice should come soon after demonstration of the skill.

(iv) The students should be certain that the children are physically and mentally mature enough to benefit from the skill to be taught.

Review Lesson Plan

The review lesson is designed to re-organize and present a different view and clear understanding of the previous learning. The following steps have to be followed in a review lesson

i. Narration
ii. Questioning
iii. Topical Outline
iv. Black Board summary
v. Tests, Questionnaires, Quiz
vi. Visual Aids, Films
vii. Excursions
viii. Assignment
ix. Performance exercise (music, drama)

These techniques are delicate that a teacher has to very careful while adopting this lesson.

Appreciation Lesson Plan

Appreciation lesson is designed to evoke an emotional or aesthetic reaction among the students by a teacher. The aim is to foster in the children a receptive attitude towards beauty and to cultivate a sense of dis-

crimination. The teacher has to keep in mind that

i. Feelings are nurtured and not taught unless the teacher himself has the attitude he desires to foster, it is unlikely that he will succeed in communicating to his pupils.

ii. Enjoyment is at the centre of appreciation. Every appreciation lesson should prove an enjoyable experience.

iii. The teacher should take care that the emotional response he wishes his pupils to make is not beyond their level of maturity and understanding.■

CHAPTER - 6

EVALUATION IN MATHEMATICS

New concept of evaluation

Evaluation is the process of assigning value to something . This is possible only on the basis of specific pre-determined goals. From the point of view of the class-room teacher, instructional objectives act as the basis of evaluation. Evaluation based on pre-determined objectives is called objective - based evaluation.

Types of evaluation - Formative and Summative

Formative Evaluation

While teaching, the content to be taught is presented in the form of small teaching points with a view to facilitate easy assimilation. At the end of each of such item, students have to be evaluated with respect to the anticipated objectives. At this stage, weakness, if any should be diagnosed and remediated. This will ensure mastery of the subject in terms of realization of educational objectives. This is known as formative evaluation.

Summative Evaluation

As the term indicates, summative evaluation is done at the end of something attempted. It may be conducted at the end of a unit, or at the end of a term covering a number of learning units. The results of summative evaluation will give a general picture of the level of attainment, in terms of instructional objectives. It may also aim at placement, prediction etc. of the learners, as in the case of annual examination or competitive test.

Comprehensive and Continuous Evaluation. (CCE)

The Comprehensive and Continuous Evaluation (CCE) provides accommodation for individual differences. It aims at fostering individual ability of children and helps them to realize their potentialities. The CCE also aims at making up the deficiency by laying adequate emphasis on the development of non-scholastic areas. Thus, it helps to develop all aspects of the child's growth to his optimum potential. Therefore, Comprehensive and Continuous evaluation presents a combination of external and internal evaluation.

Objectives of Comprehensive and Continuous Evaluation

The main objectives of CCE are as follows:

- To foster individual abilities of the children.
- To help the children to realize their potentialities and capacities.
- To enable teachers to evaluate those attitudes, abilities and skills which are impossible to evaluate through traditional examinations.
- To emphasis the development of non-scholastic areas.
- To help children to have periodical feed back to

judge their achievements.

- To provide remedial and enriched instructions.
- To evaluate comprehensively the more important abilities like affection, attitudes, interests, habits etc.

The areas of Comprehensive and Continuous Evaluation

The scheme of CCE covers the following aspects of personality of a child:

- Academic achievement of the child.
- Personal and social qualities such as regularity, responsibility, punctuality, cleanliness, co-operation, initiatives, sense of social service etc.
- Interest in cultural, artistic, literary, scientific endeavours.
- Health Status.

Tools and Techniques for Comprehensive and Continuous Evaluation

Various techniques such as interview, observation and tools like check lists, rating scales, etc. can be used for evaluation of non-academic areas. Various techniques for performing comprehensive and continuous evaluation are:

Quizzes
Assignments
Written and Oral tests
Practical Work.
Term papers
Seminars / Group discussion
Weightage to attendance

Achievement, Diagnostic and Prognostic Tests

The process of instruction involves three important tasks namely teaching, learning and evaluation. By continually assessing the achievement of the instructional objectives by the students, the effectiveness of the learning experiences provided and the instructional strategy used by the teacher can be assessed. Though a teacher may have to use techniques and tools such as interview, observation, case study, cumulative record, rating scale, check list etc. tests constitute the most important means to evaluate student's performance.

The present system of instruction is based on predetermined objectives. The teacher has to find how far the objectives have been achieved. In order to assess the degree of realization of the objectives by the learner, the teacher has to conduct a test which is known as an achievement test. Why a particular learner could not achieve the objectives, can also be determined in terms of gaps and difficulties. A test meant for that purpose is a diagnostic test. The future prospects of success of a student in any selected area can be predicted by giving a test designed for that purpose. Such a test is known as a prognostic test.

Achievement test

Every teacher wants to find out the progress made by his pupil in the subject he teaches. Achievement in a subject at a particular stage has to be assessed in terms of his mastery in the curricular provisions anticipated for that stage as well as the realization of the objectives expected. A test designed to assess the achievement in any subject with regard to a set of predetermined objectives is called as an achievement test. The International Dictionary of Education defines achievement test as a "test designed to measure the effects of specific teaching or training in an area of the curriculum".

Functions of an achievement test.

- To help in determining the placement of students in a particular section.
- To help the teacher in identifying pupil's difficulties.
- It provides a basis for promotion to the next grade.
- It helps the teacher to see for himself / herself how effectively he/she could transact the curriculum.
- To motivate the students before taking up a new assignment.

Achievement Test— Construction

The concern of a test- designer is to see how all the qualities of a test can be maintained to the maximum possible, within the limitations of the environmental conditions available.

Planning

The first step is to determine the maximum time, marks and nature of the test. These should be decided in terms of the nature and scope of the unit or units involved in the testing. A test for a single unit may be generally of forty or forty five minutes duration, with a maximum of twenty to twenty-five marks. But in the case of a test conducted at the end of a term, a semester or a session the duration may be about two hours and maximum marks may be 50 or 100.

After determining the broad scope of the test, a

design has to be developed. The objectives, content, forms of question and the weightage to difficulty level are the most important factors to be considered in such a design. What is required is to analyze the syllabus in terms of the objectives and the content area and determine the relative weightage to each of the pre-determined objectives as well as the sub units into which the content have been divided. In the same way the weightage for the different forms of the questions to be included and for the difficulty levels to be maintained also are considered while finalizing the design. This will be followed by the scheme of option and the scheme of sections into which the test has to be divided if required.

Design of a test

Designing the test is the most important step in test construction. Following decisions have to be taken

(i) Weightage to instructional objectives

After identifying the objectives to be tested, relative weightage has to be given for each objective. Weightage to be given in numerical terms, giving the greatest number to those that are to receive the greatest emphasis based on the nature of content.

S. No.	Objective	Mark	Percentage
1.	Knowledge	8	16
2.	Understanding	20	40
3.	Application	10	20
4.	Skill	10	20
5.	Interest	2	4
	Total	**50**	**100**

(ii) Weightage to content area

Content being the means through which objectives are to be realized, it is essential to fix the weightage to be given to its different parts. This has to be done considering the factors like how important is the sub. unit for the total unit, how much time was allotted for its instruction and the length of the subunit.

S. No.	Sub unit	Mark	Percentage
1.	I..........	15	30
2.	II	10	20
3.	III..........	10	20
4.	IV	5	10
5.	V	10	20
	Total	50	100

(iii) Weightage to forms of question

The test maker has to decide the forms of question, number of questions to be included in each form and the relative weightage.

S.No.	Form of Questions	No. of Questions	Mark	Percentage
1.	Objective-type	25	25	50
2.	Short-answer	5	15	30
3.	Essay	1	10	20
	Total	31	50	100

(iv) Weightage to difficult level

The difficulty level of the items has to be decided based on the group of students for whom it is designed. To get optimum discrimination through a test, most of its questions should be of average difficulty level. A few easy questions to motivate the below average students and a few difficult questions to challenge the above average should find its place.

S. No.	Level of difficulty	Mark	Percentage
1.	Easy	10	20
2.	Average	30	60
3.	Difficult	10	20
	Total	50	100

(v) Scheme of option

There will be no option for objective - type and short answer. For essay questions options can be given.

(vi) Scheme of sections

The test will be in two sections A and B. Section A will contain all the objective - type items to be answered in the test itself. Section B will be meant for short - answers and essay questions to be answered in separate answer book.

Blue - Print for a test

The next important step in the construction of an achievement test is preparing a blue print according to the design. A variety of blue prints can be prepared according a single design. This can be done by arranging the questions under each category of objectives, content etc in different ways. A 'typical' blue-print is prepared as a three dimensional chart, indicating the distribution of questions, objective-wise, content-wise and form-wise. It can be made four dimensional including difficulty level as one of the dimensions.

Note:

(i) O - Objective - type

S - Short answer

E - Essay

(ii) The numbers outside the brackets indicate marks and those inside indicate the number of questions. (see page no. 105)

Objective	Knowledge			Understanding			Application			Skill			Interest			
Forms of question / sub unit	O	S	E	O	S	E	O	S	E	O	S	E	O	S	E	Total
I	2 (2)	3 (1)				10 (1)										15
II		3 (1)		4 (4)					3 (1)							10
III							5 (5)			2 (2)	3 (1)					10
IV				3 (3)									2 (2)			5
V					3 (1)		2 (2)			5 (5)						10
Sub Total (2)	2 (2)	6	(7)	7 (1)	3 (1)	10 (7)	7		3 (1)	7 (7)	3 (1)		2 (2)			
Total	8			20			10			10			2			50

In distributing the marks in the blue-print, the nature of the content has to play a key role. If a particular sub-unit involves more facts than concepts and principles it will be advisable to have more knowledge items from this sub-unit rather than items involving complex behaviour.

Organisation of the test

After finalysing the items, these have to be arranged according to the scheme of sections indicated in the design. For arranging the items, psychologically, it is advisable to arrange in the order of difficulty level. The hierarchical order of the objectives in the taxonomy is an indication of the difficulty level also.

Scheme for evaluation

One of the steps suggested for maintaining the objectivity is to make the scoring strictly in accordance with a pre-designed scheme of evaluation. In the case of objective type item where the answers are in the form of symbols or letters a 'scoring key' showing the number of questions and its correct answer (key) can be prepared as follows.

Scoring key

Question number	1	2	3	4	5
Key	A	C	D	B	

For short-answer and essay questions, the scheme of evaluation is different. A short answer questions may contain one to three value points. Each of these value points should be weighted at the time of scoring. It is not necessary that all the value points are allowed the same weightage. It depends on its nature,

difficulty etc. The 'making scheme' in short-answer and essay type is as follows.

Marking Scheme

Question number	Value points	Score for each point	Total
6	(i)	½	
	(ii)	½	2
	(iii)	1	
7	(i)	1	
	(ii)	1	2

Preparation of a good scheme of evaluation may even help in refining the test.

Question - wise analysis

A final and detailed analysis of the test is attempted before releasing it. This is done by making an analysis of each item, considering all the aspects from objective to the time required. It can be prepared as shown below.

Question number	Sub unit	Objective	Specification	Forms of question	Difficulty level	Marks	Times in minutes
1.	I	Knowledge	Recognizes	Objective	Easy	1	1
2.	III	Understanding	Identifies the relationship	Short answer	Average	2	3

A scrutiny of this table will finally show whether all the aspects envisaged in the design and blue-print are satisfied by the test in its final form.

Diagnostic test

The learner's progress is to be appraised in terms of desirable educational objectives. Therefore it is necessary to identify deficiencies in the level of achievement together with reasons for these deficiencies. It they are not identified then and there, they might accumulate and after a stage learning might become impossible. So a teacher is required to know the specific weakness of the pupils both individually and collectively and has to take suitable remedial measures. The test designed to locate the specific weakness in the learning of the pupils is known as diagnostic test. It measures how much a learner has not been able to achieve and why. The need for diagnostic testing arises in mathematics specifically at a time when a particular learner exhibits some signs or symptoms of his failure or difficulties with regard to the learning of the subject.

Diagnostic testing in mathematics can be defined as a testing or evaluation programme carried out by a mathematics teacher for diagnosing the nature and extent of the learning difficulties and behavioural problems of the learners along with the inherent causes for chalking out suitable remedial programmes aimed to help them in getting rid of their difficulties and problems.

Remedial teaching

Diagnostic testing and remedial teaching are interrelated and complementary to each other. The scientific use of a program of diagnosis is an important aspect of functional teaching. Its real value depend upon a follow

- up program of remedial teaching and a careful check on and interpretation of attained results. There is however, no set pattern for remediation. In some cases, it may be a simple matter of review and reteaching. In others, an extensive effort to improve motivation, correct emotional difficulties and to overcome deficiencies in work and study habits may be required. Lack of prerequisites to learn new material also may stand in the way of proper learning. For two students the learning difficulty may be same but the causes can be different. So remedial programmes will differ from individual to individual. To be most effective the remedial material should possess the following characteristics:

1. It should be selected to bring about certain definite ends.
2. It should be on component elementary skills.
3. It should be capable of self-administration by the student.
4. It should be of such a nature that it could be administered to a class, to an individual, or to a group.
5. It should be correlated with the instructional material being used.
6. It should be provided with answers.

Construction of Diagnostic test

To start with, the content of the unit for which the test is envisaged should be thoroughly analysed, first into the teaching points involved. Each of these points may include a number of stages and these may then identified and arranged in the sequential order of difficulty. The next step is to write test items representing all the minute steps arising out of the analysis and these items may then be arranged in the

order, taking into consideration both sequence of the stage and difficulty level. While constructing a diagnostic test, it will unpshychological to make it too long and hence in such cases the set of items may be divided into two or three forms to be answered on different occasions.

Prognostic test

A test which predicts the future performance of a student in a particular area is known as a prognostic test. As they usually test the background skills and abilities found to be prerequisites for success in the particular subject, prognostic tests are more common among subjects in which success can be rather well defined in terms of certain basic abilities. They also frequently test some of the aptitude factors that are not directly dependent upon previous training in a specific area. So prognostic tests are closely related to aptitude rather than to other aspects like intelligence.

Prognostic tests can be used to reduce the number of failures either by eliminating those who are unable for any cause, to proceed further with mathematical study or by providing a basis for the construction of a differentiated mathematics curriculum. Such tests serve as an aid in the vocational and educational guidance of pupils and in the better classification of pupils.

The inventory test is used for the purpose of "taking stock" of mathematical information and ability. It should show what a student knows about a certain topic. Under the modern philosophy of mathematical education the student has many opportunities to learn something of elementary algebra and a good deal of instructive geometry by the time he enters the secondary school. As he proceeds up the instructional ladder, seasonal inventory tests will prevent a great deal of unnecessary repetition of experience on the part of the students and waste of effort on the part of the teacher. Such tests may also be used effectively to bring to light the background which the students have for study of new units and thus aid in the guidance program.

The discovery of superior ability and unusal aptitude in mathematics is just as important a function of prognosis as is the discovery of the inferior or average. For the construction of efficient prognostic tests in mathematics the teacher should be familiar with those abilities and interests essential to further progress. The general characteristics of comprehensiveness, discriminative power, reliability, validity, balance, and flexibility must then be carefully observed in the framing and organization of the test items.

Qualities of a good test

While constructing a test the following criteria should be observed with utmost care.

1. A test should be as highly objective as possible

The element of personal interpretation should be minimized in the determination of the correctness or incorrectness of student reactions to behaviour situations

2. A test should be reliable

The reliability of a test is determined by the consistency with which it measures that which it does measure. The behaviour of the examiner ,the mental and physical condition of the student, and the condition under which the test is given have a great deal of influence upon the reliability of the results obtained from any test.

3. A test should be valid

If a test is valid, it is valid for a given purpose, with a given group of pupils, and is valid only to the degree that it accomplishes that specific purpose for that specific group. The significant attributes of validity are reliability and objectivity. But they do not guarante the validity of a test. To be valid the test must be further characterized by comprehensiveness and discriminative power.

4. A test should be economical of the teacher's time

The amount of time required for the construction, administration, and interpretation of a test should not be excessive.

5. A test should be 'student conscious'

The elements of the test should be couched in non-ambiguous language, and reasonable tasks should be set for reasonable periods of time. In test items designed to measure understanding of a principle or ability to apply a principle, computation should be minimized.

6. A test should motivate the best efforts of the students

The question should be so worded and presented that they will discourage guessing and bluffing. A test should never be used as a means of punishment but should always tend to create in the mind of the student the attitude that it is worth taking.

7. A test designed to discriminate between student's abilities must provide for measurement of the entire range of abilities

If an accurate discrimination between abilities is to be approximated, there must be questions easy enough that all students can answer them and questions so difficult that perfect scores would be highly improbable, if not impossible. Some questions should be so designed that the student will have the responsibility of distinguishing between essential and non-essential data.■

CHAPTER - 7

MATHEMATICS CURRICULUM

Curriculum is the crux of the whole educational process. The term curriculum is derived from the Latin word *Curee* which means 'a path to run'. So curriculum is a path or way over which a child runs to achieve the aims of education. A wider sense of curriculum signifies all those activities and learning experiences which a child undergoes in and outside the class according to his needs, attitudes and interests.

According to Secondary Education Commission or Mudaliar Commission Report (1952-53), "Curriculum does not mean only the academic subjects traditionally taught in the school, but it includes the totality of experiences that pupils receive through manifold activities they do in the classroom, library, laboratory, workshop, playgrounds and numerous informal contacts between teachers and pupils".

Definition of Curriculum

Arthur Cunningham ham defines "Curriculum is a tool in the hands of the artist (teacher) to mould his material (pupil) according to his ideals (objectives) in his studio (school)".

In the words of Crow and Crow "Curriculum includes all the learners' experiences, in or outside school that are included in a programme which has been devised to help him to develop mentally, physically, socially, emotionally, spiritually and morally".

Mathematics curriculum is a pivot on which the whole process of teaching - learning involves. It provides the necessary insight to the mathematics teacher in the selection of the learning activities, teaching methods, learning resources and evaluation techniques.

Major objectives of Mathematics Curriculum

The Mathematics curriculum aims at the following objectives:

- Proficiency in fundamental mathematical skills.
- Comprehensiveness of basic mathematical concepts.
- Appreciation of significant meanings.
- Development of desirable attitudes.
- Efficiency in making sound mathematical applications.

The curriculum planning involves two major stages:

(i) Curriculum Construction

(ii) Curriculum Organisation

Principles of Curriculum Construction

While developing mathematics curriculum the content should be selected according to the changing needs of society in general and the subject, in particular. The following principles should be kept in mind while constructing the mathematics curriculum.

1. Principle of Utility

While constructing a curriculum in mathematics, topics which are useful to the target group from a utility point of view should be included. These topics should be:

- Helpful in day-to-day life.
- Helpful in learning other subjects.
- Helpful in providing basis for many vocation.
- Helpful in proper understanding and progress of ones culture and civilization.
- Helpful in realizing the aesthetic value of the subject.
- Helpful in understanding the scientific and technology progress.

2. Principle of Community Centeredness :

A curriculum should serve the community of a particular place by educating the children according to the needs, aspiration and ideals of that community and therefore be constructed and shaped for the welfare of the local community.

3. Principle of Flexibility

A curriculum should be flexible in nature so that it can be modified and reshaped according to the circumstances and demands of the resources in hand. In this context the emergence of topics like Topology, Matrices, Linear programming may be cited as example.

4. Principle of Child-centeredness

While deciding the contents for the curriculum, child's interest, abilities and age level should be kept in mind. If the pupil has to reveal initiative, co-operation and social responsibility these qualities should be developed in them by means of meaningful activities appropriate to their stage of psychological development.

5. Teacher's point of view

While framing or revising the curriculum, the teachers who are the real field workers, who are to transact the curriculum should be considered. They know the level of students and the items which should be and could be taught to the students.

6. Principle of Disciplinary value.

Mathematics has disciplinary value as it disciplines and trains the faculties of mind like reasoning, thinking, imagination, memorization, concentration, inventiveness etc. Topics which help in the task of disciplining the mind should be included in the curriculum. It is proved that real useful problems of mathematics train the mind better than the unreal fancy problems like riddles, puzzles and catch problems.

7. Principle of Cultural value

The subject mathematics has played a great role in the advancement of culture and civilization. There may be certain ideas, which were pursued in olden days by the students of mathematics, but of no use now. But there are certain ideas and facts in mathematics, which still form an integral part of modern culture and society. Such ideas should be included in the curriculum. Its study will be a source of inspiration to the students to preserve, transmit and renew culture.

8. Principle of Correlation

While organizing content in mathematics, topics should be included in such a way that it can be correlated in the following ways:

- Correlation with life.
- Correlation with other subject.
- Correlation among different branches of the same subject.
- Correlation among the topics of the same branch.
- Correlation with work experience.

9. Principle of Preparatory value

Education at one stage must aim to prepare the child for education at higher stages. Therefore curriculum at any stage must cater to the needs of the higher classes. At the same time preference should be given to school leavers to equip them with all the essential knowledge of mathematics to lead a balanced life.

10. Integrating theory with practice

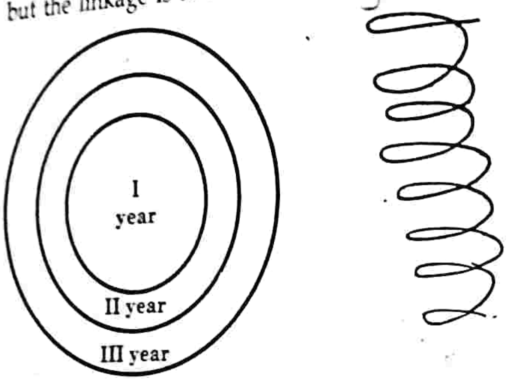
Theoretical knowledge without its practical application and vice versa are dangerous. Mathematics curriculum requires topics, contents, experiences and activities in such a way that there is enough opportunities for integrating theory with practice.

Approaches to Curriculum Organisation

After topics have been selected according to the relevant fundamental principles, they have to be systematically arranged so as to facilitate meaningful and effective transaction. The important among them are:

1. Concentric and Spiral approaches

(The whole curriculum is spread over a number of years. In the beginning of the course, all the essential aspects are given to pupils in a simplified way. In the coming years more and more of its parts are added. It follows the maxims of teaching such as from whole to part, simple to complex, easy to difficult etc. This approach is referred to as concentric approach. The term spiral gives the additional implication that while attempting gradation the linkage too is taken care of and the continuity of the topic concerned is never broken. In concentric only the widening of the scope is indicated, but the linkage is not taken care of.



Concentric Aapproach Spiral Approach

Psychological and Logical approaches

The arrangement of subject matter based on the principles of psychology is known as psychological approach. In this, the criterion for inclusion of an item will be psychological needs, requirements, potentials, capacities etc. appropriate for the developmental level of the stage for which the curriculum is being designed. In logical approach stress is given to the logical sequencing of the subject concerned. It is criticized that by splitting topics to suit the developmental status of the learner, the logical sequence is broken. Logical approach demands maintaining the logical sequence while developing the curriculum. A good curriculum, if carefully developed

can maintain the psychological approach without sacrificing the logical sequence of the subject.

Topical and Unit approaches

Every subject of study involves a number of topics. A topic is a comprehensive collection of related learning materials pertaining to specific area of the subject, systematically and sequentially arranged so as to get a holistic picture of those aspects. There are a large number of concepts, principles, processes, and skills associated with an area, which act as related parts of the 'whole'. Since these aspects are interrelated and as they maintain certain logical sequences and correlations, it is advisable that the topic should be thoroughly dealt with and mastered before passing on to another topic. This is known as 'topical approach' in curriculum.

A topic may be so complex and might include a large number of items of varied difficulty and on such occasions, it is advisable not to cover all at the same time. A spiral approach may be adopted and study of the topic may be distributed over a longer time span. For this, the topic may be divided into a number of units. Though a unit may be only part of the same topic it can be given a hoslitic unity by properly linking the closely knit ideas involved. Taking fundamental units of the topic first and then gradually taking other units one by one in due course will make learning more psychological. This approach is known as 'unit approach'.

When a topic is complex and very large and involves units posing varied levels of difficulty it will be advisable to have the unit approach. But care should be taken to effectively link all the units of the same topic as and when opportunities arise.

Integrated approach

The purpose of education is acquisition of knowledge and its application for the study of other subjects and for solving problems in everyday life. The knowledge and skills acquired through the subjects taught without highlighting its application become meaningless. While teaching any subject, the teacher can give examples to show that knowledge is a single integrated whole. Such an integrated approach helps the students to get a holistic view of the entire school programme and thereby the study of each subject becomes more meaningful and significant.

Major reforms in Mathematics Curriculum

SMSG (School Mathematics Study Group)

This is an organization of secondary school teachers in America which was founded in 1958. Under the direction of Edward. G. Begne the aim of this organization was to make the curriculum up-to-date and to acquire a place in the world map of revised mathematics curriculum. The SMSG has made a very critical analysis of the different curriculum of the various school systems with a view to make a new and up to date curriculum. They published a series of mathematics text books for the students of both junior and senior schools. A parallel series of handbooks for teachers were also published. The SMSG has given due consideration to the new trends in the construction of mathematics curriculum. Most of the modern topics were given a place in the SMSG books. In these books the structure of mathematics was given more stress than content area. To a great extent the books were challenging to the pupils. Proper attempts have been made in grading exercises which help the pupil in self-study. Size of the book was not appreciated much. The pattern of curriculum revision of SMSG was

unique, that it began from grade 7 though 12, then went to 4 through 6 and finally the kinder garten through grade 3. SMSG completed its set of text books in 1966 for the entire range. SMSG produced programmed text books, special books for teachers, film strips etc.

Yale University U.S.A has published a set of 16 books prepared by School Mathematics Study Group (SMSG). Eight of these books are text books for use in Secondary Schools and eight companion volumes are teacher guides. Four books deal with arithmetic, two with algebra and two geometry.

Key ideas underlying these books are structure of arithmetic from algebraic view point, real number system as a progressive development, metric and non-metric relations in geometry, appreciation of abstract concepts, the role of definition, development of precise vocabulary, experimentation and proof. In contrast to traditional beginning of text-books in arithmetic the opening chapter of SMSG is 'Mathematics as a method of Reasoning'. Algebra is introduced through 'sets'. Similarly the stress in geometry is on explanation of concepts.

Based on this modern concept of mathematics curriculum as practiced in U.S.A. the National Council for Educational Research and Training (NCERT) has developed some text books in Mathematics for use in Secondary schools in India. The schools in Delhi were among the first to adopt this modern mathematics in their school curriculum. A more balanced curriculum is now being adopted in the national (10+2) pattern.

SMP (Scottish Mathematics Project)

In 1961, the mathematics curriculum was reconstructed in Scotland with the intension of teaching mathematics heuristically in schools. They prepared five text books for the pupils studying for GCE (General Certificate of Education) examination at 'O' level. Later in 1967 they prepared a series of text-books from A to H levels for the students studying for CSE examination. Hand books for teachers were also prepared.

Nuffield Mathematics Project (NMP)

This was set up in 1964 by the Nuffield foundation of England to develop new mathematics teaching for children. The project was designed for children of age group 5 to 13. According to this group children of age group 5-13 must be set free in order to make their own discoveries and in this way to achieve understanding. The title of the first volume in the Nuffield series of publication is 'I do I understand'. Readily available materials are preferred to expensive devices. It emphasized child centered learning and stressed the need for interaction of the children in exploring situations. NMP has developed appropriate text books, published teacher guides and necessary instructional materials.

NCERT (National Council For Educational Research and Training)

In India, NCERT reconstructed mathematics curriculum based on modern concept of mathematics curriculum as practiced in U.S.A.

According to NCERT National Curriculum Frame Work (NCF) 2005, the structure of curriculum should be as follows.

At the pre-primary stage, all learning occurs through play rather than through didactic communication. Making simple comparisons and classification along one dimension at a time, and identifying shapes and symmetry, are appropriate skills to be acquired at this stage.

Helping children to develop a positive attitude towards the subject using mathematical games, puzzles and stories is important.

At the upper primary stage, students come across with the application of powerful abstract concepts. Data handling representation and interpretation form a significant part of the ability, which is an essential 'life skill'. Learning at this stage also offers an opportunity to enrich students' spatial reasoning and visualization skills.

At the secondary stage, students begin to perceive the structure of mathematics as a discipline. Individual and group exploration of connections and patterns, visualization and generalization, and making and proving conjectures are important at this stage. The students integrate many concepts and skills that they have learnt into problem solving ability.

The aim of the mathematics curriculum at the higher secondary stage is to provide students with an appreciation of the wide variety of the application of mathematics, and to equip them with the basic tools that enable such application.■

CHAPTER - 8

MATERIALS FOR TEACHING- LEARNING MATHEMATICS

Mathematics Text Books

The text book plays a key role in effective teaching and learning. Now a days text book has become a course of study, a set of unit plans and a learning guide as well. A text book should really be designed for the pupils than for the teacher. So it should stimulate reflective thinking and develop problem solving ability among the students.

Uses of a text book in Mathematics

A mathematics text book is useful for the teacher and students in the following ways

I. A mathematics text book helps the teacher in

- Deciding the outline of the course content, planning the lesson, selecting problems to be solved, teaching aids to be used and methods to be adopted for each lesson.
- Presenting the subject matter in a systematic way.

- Providing individual attention to the learners.
- Giving drill and home work to the learners thereby developing their speed and accuracy.

II. A mathematics text book helps the learners in

- Getting acquainted with what is to be learnt in a particular class.
- Filling the gaps while understanding a concept.
- Fostering the right study attitude.
- Saving time and energy as they need not have to copy down problems.
- Moving at their own pace.

III. A mathematics text book helps in maintaining uniformity of standard in different schools of a district or a state. This uniformity serve the purpose of common curriculum and evaluation

Characteristics of a good text book in Mathematics

- Text-book should be written by competent and experienced teachers.
- It should be written in line with the objectives of teaching mathematics of a particular class.
- It should cater to the needs of slow, average and fast learners.
- The content of the book should be up-to-date.
- The subject matter must be organized according to the psychological as well as logical order.
- All the progressive methods of teaching mathematics like indo-deductive, analytic-synthetic, problem solving and heuristic methods should find its place in the text book.
- The examples given should not be too much or too less in number.

- Definitions, concepts and principles given must be clear and definite.
- There should be provision for the revision of the work done in the previous class.
- Adequate number of exercises should be provided for each lesson.
- Symbols and formulae given should be of national standard.
- Illustrations in terms of pictures and diagrams should be appropriate.
- Oral mathematics should find its place in the text book.
- The language used must be simple and clear.
- Valuable suggestions should be given in the text book for projects, assignments and correlated learning.
- The paper used should be of good quality.
- The size of the text book should be handy.
- A text book should be attractive in appearance.
- The text book should be free from errors.
- It should be moderately priced.

Work book in Mathematics.

For effective learning, pupils should have opportunities for practicing what they have learnt. Pupils' work book is a practice material in which the content is arranged in the order of the text book on which it is based. A work book contains summary of each chapter of the text book, exercises of different kinds, key to exercises and suggestions for practical, assignments and improvisation.

Function and use of work book

The function of the work book in mathematics is more specialized than that of the text book. Work books are generally used as supplementary teaching devices. They are built with the idea of suggesting appropriate activities and exercises rather than with the idea of providing information. Workbooks are merely specialized and elaborated forms of a teaching device that has been in use as long as mathematics has been taught from books, viz, the suggestion of problems and activities through which it is expected that students will acquire the desirable skills, understanding, abilities, and appreciations which are the objectives of the work.

The work book eliminates the necessity for copying problems and exercises, and in this way it saves time for the students and prevents them from making mistakes that may have no relation to their ability to perform the required mathematical tasks. It provides a uniform pattern for the written work of all the students.

Characteristics of a good workbook

A mathematics work book should embody the following characteristics

1. It should aim to be instrumental in bringing about certain definite educational outcomes. These outcomes should be specified in the workbook in its preface.
2. Space should be provided for most of the written work to be done in the book itself.
3. The content should supplement that of the basic text book.
4. It should economize the students' time by minimizing the need for copying exercises.

5. It should economize the work of the teacher in checking the written work.
6. It should provide opportunity for each individual student to work at his own optimum rate.
7. It should provide some arrangement whereby each student can keep a record of his own achievement and progress.
8. It should be accompanied by a list of answers to facilitate checking.
9. It should have a pleasing format and the type face should be large enough to be easily read.
10. It should be attractive in appearance and moderately priced.

There are work books combined with text books. They are not just text books with the work books attached as appendices. The two are woven together and co-ordinate at all points. The main difference between these books and the usual text books is that in these combination books the exercises are generally put up in work book form with space provided so that most of the written work can be done in the book rather than on separate paper. Its disadvantage is that there is no scope for resale or reuse.

Guide Books for Teachers

It is a reference material for the teacher to supplement his classroom teaching. It contains summaries of chapters in the text book, statement of the instructional objectives to be realized, explanation of significant terms, facts, principles, learning experiences to be provided, demonstration exercises of various kinds suited to each topic, evaluation tools, assignment for pupils etc. It suggests the teacher the measures to be adopted to

make his instruction suitable and effective in the classroom situation.

Advantages

1. It equips the teacher with sufficient theoretical knowledge about what is to be taught.
2. The problem of finding out suitable learning experiences for teaching different lessons is easily solved.
3. It helps the teacher to adopt appropriate methods to teach every topic.
4. It helps the teacher to evaluate pupils using relevant evaluation tools.■

CHAPTER - 9

THEORIES OF EMINENT PSYCHOLOGISTS

Education is a productive process and technology has its impact on teaching-which is curriculum transaction. The product expected from curriculum transaction is development of the learners that has comprehensiveness, substainability and transferability. If this goal is to be realized the mathematics teacher has to apply appropriate technology for curriculum transaction. Instructional technology has been designed on the basis of learning theories formulated by educationalists.

A. Jean Piaget

Jean Piaget conceives learning as information processing by which the learner through active interaction with his fellow learners as well as the teacher independently goes on developing the cognitive structure. Hence he is considered as a cognitive develop mentalist or a constructivist. He visualizes the class as a shared environment that facilitates thorough and meaningful learning.

Cognitive Development

The basic concepts involved in the theory of cognitive development are:

(i) Schema

Schema is the basic element using which the cognitive structure is built up. It is a mental image formed by the learner from experience. Processing of newer information is possible only if all the related schema are meaningfully internalized. When a series of schema are chained together to describe a complex process it becomes a 'Scheme'. The mathematics teacher has to provide experiences to the learners to create as many schema as possible and also to ensure the quality by making them meaningful to the learner. To construct a meaningful understanding of a circum circle of a triangle, meaningful understanding of a large number of schema that are linked together in the correct order is required. The total image gained about the process is a scheme.

(ii) Disequilibrium

Every organism would like to maintain a level of equilibrium in the conditions required for the satisfaction of basic needs. If these needs get disturbed the result will be disequilibrium that might create anxiety to the organism. When a learner becomes curious to know about something, his intellectual equilibrium gets disturbed. The learner will try hard to regain the lost equilibrium. Hence Piaget says that in classroom the teacher has to create intellectual disequilibrium among the learners so that they will try to get equilibrated by finding out a satisfactory solution to the problematic situation. This involves processing of information which in turn result in the development of the cognitive structure of the learners.

(iii) Assimilation and Accommodation

When an unfamiliar situation is presented, the learner looks back for his previous experiences on the basis of which the newness can be eliminated. When solution is arrived, the learner gets equilibrated. This will help the learner to adapt with the situation. The process of making an unfamiliar situation familiar with the help of some familiar schema that would help in regaining equilibrium leading to adaption is called assimilation.

If the new experiences are to be converted into a meaningful schema so that it is made a part of the cognitive structure, the learner has to find out an appropriate place in the existing cognitive structure for filling the new experience. This will enable him to logically accommodate the new schema which has been internalized by assimilation. This process of registering the new schema in an appropriate place in the existing cognitive structure is known as accommodation. As a result of such accommodation the cognitive structure gets developed.

Stages

As children advance in age, their ability for operation goes on increasing and hence the nature of cognitive development is determined to a large extent by the age of the individuals concerned. Piaget has formulated four different developmental stages.

i. The sensory motor stage (birth to 2 years)

The starting point is purely reflex action like turning the head towards a toy that produces sound. Gradually the child develops the ability to grasp something with the fingers.

ii. Pre-operational stage (3-7 years)

This stage is divided into two (a) intuitive stage and (b) pre-conceptual stage

During the first stage the child performs certain actions intuitively, without any idea of the logic behind it. During the second stage the ability for concept formation begins.

iii. Concrete operational stage (7-11 years)

The child during this stage develops the power for operation. Logical thinking is possible, but only in situations presented in a concrete manner. A mathematics teacher can aim at giving complex schema but the operation will be based on concrete experiences only.

iv. Formal operational stage (11 years and above)

During this stage the ability for mental operation blossoms to full and children can relate a number of ideas and draw conclusions logically even in the absence of concrete experiences.

Implications

First of all the mathematics teacher should study the nature of each developmental stage and the learning activities should be planned according to the developmental stage of the learner. Then the teacher should pose before the child challenging problematic situation so that his intellectual equilibrium is disturbed and he will be alert for facing the challenge. The teacher can guide the student in the process of linking the new experiences with familiar ones and thus gaining equilibrium leading to adaption, but the responsibility for constructing the knowledge and systematically developing the cognitive structure should be with the learner.

B. Jeorme. S. Bruner

Bruner believes in the constructivist theory regarding information processing leading to development of cognition. He is of the opinion that the learner should discover knowledge by himself. Discovery learning helps the learner learn how to learn. Process of learning is considered more important than what is learnt.

Strategies for the teacher

i. Spiral approach

If the learner has to discover and construct knowledge on his own, the planning of the curricular experiences and the activities involved in curriculum transaction should maintain gradation. The teacher has to make maximum use of the spirit of discovery possessed by the learners as evidenced by the strong curiosity enhibited by them. Gradation involves curricular experiences. The teacher has to follow spiral approach both in the development of the curriculum and the strategies adopted for transacting the curriculum.

ii. Gradation of learning activities

Bruner suggests gradation in learning activities also. The activities should provide three level of experiences to the learner viz. (a) enactive (b) ikonic and (c) symbolic.

The first type involves physical action and activities in which the learner has to enact. For example, to teach the concept of number 3, three fingers can be demonstrated. Whereas in ikonic experience, mental images are formed. Five apples or five pencils can be shown. Then the learner can be given symbolic experiences by describing the number using language – two and one make three, one taken from four gives three and so on.

Bruner's theory of concept formation

According to Bruner a concept is a mental image of a set of objects or any phenomenon that could be classified into a 'set', all members of which obey a set of attributes. For example, the concept of 'triangle'. It represents a generalised class of geometrical shapes all of

which obey a set of attributes. Triangles of various types and sizes and the common attributes like three sidedness and closedness.

Steps in concept formation

Concept formation involves a number of hierarchical steps:

i. Observing a number of exemplars and non exemplars.

ii. Analysing the attributes of each item.

iii Comparing exemplars and non-exemplars according to their attributes.

iv Classifying exemplars and non-exemplars according to their attributes.

v. Arriving at a definition based on the attributes.

vi. Examining further cases to reinforce the definition arrived.

Mathematics is a subject that deals with innumerable concepts and hence application of Bruner's idea about internalizing concepts is very crucial for a mathematics teacher.

C. Gagne

Gagne's contribution is 'hierarchy of learning' in which he explains how the nature of the learning material can be arranged hierarchically so that the learner can be exposed to the appropriate learning experience at each level. He stressed the need for ensuring thorough mastery at each stage and chaining the experiences. Then only the learners can apply the learnt material in new situation.

The hierarchy of learning

Gagne has identified eight steps in the learning hierarchy. They are

i. Sign learning

This points out to the stage at which there is fixed response for each sign.

ii. Stimulus-response learning

In this type of learning the response is more meaningful. What the teacher has to do at this stage is to help the learner identify an appropriate response for a given stimulus and to reinforce it.

iii. Chaining of motor activities.

At this stage the learners are made to establish chained relations by linking many stimulus - response bonds.

iv. Verbal association

This also involves meaningful chaining of a number of actions. But it is different in that the chaining is done using language. In order to do this the learner has to be definite about the meaning of each word involved. This stage is very important because efficiency in making verbal association is a stepping stone to concept formation.

v. Multiple discrimination

In this the learner engages in a number of meaningful responses in a given situation which involves a variety of stimuli. For example the number eight can be described as combining two fours, one less than nine or three added to five etc.

vi. Concept learning

This is the type of learning in which different phenomena are compared on the basis of attributes and a generalised image is formed. Closely observing a rhombus, noting down its attributes, comparing with other quadrilateral will help the learner arrive at a generalised concept.

vii. Rule learning

When two or more concepts are meaningfully linked, a principle is arrived. For example, a theorem or a formula. What is required in rule learning is not to learn by heart the principle, but to internalize the concepts involved and meaningfully linking them.

viii. Problem solving

In mathematics, the major activity is problem solving. It is a process that involves meaningful and appropriate application of principles in the correct order, which results in a new product viz. solution to the problem. This stage which is the most complex of the learning items could be mastered only if the previous stages are mastered.

Implications

The most significant implication of the hierarchical stages is the need for mastery of pre-requisites to learn a particular item. If the pre-requisites are not achieved one cannot master the expected knowledge or skill. In other words, concepts and principles act as the pre-requisites for mastering problem solving.■

CHAPTER - 10

MODELS OF TEACHING

A model of teaching is just a blue print designed in advance for providing necessary structure and direction to the teacher for realizing the stipulated objectives.

According to Joyce and Weil, a teaching model is a pattern or plan which can be used to shape curriculum or course, to design instructional materials and to guide a teacher's actions. It also creates necessary environment which facilitates the teaching process.

Characteristics of teaching models

Models of teaching:

- are some sort of plans or guidelines or patterns or strategies of teaching.
- are systematic procedures to modify the behaviour of the learners.
- specify the learning outcomes in terms of observable and measurable performance of students.
- specify in definite terms the environmental condition under which a student's response should be observed.
- specify the criteria of acceptable performance expected from the students.

Functions of models of Teaching

Models of teaching have three major functions in the teaching learning process. These are:

(a) designing of curriculum or course of study.

(b) development and selection of instructional materials.

(c) guiding the teacher's activity in the teaching-learning process.

Generally a teaching model is described with some fundamental elements. They are:

Focus

Focus is the central aspect of a teaching model. For what the model stands is the theme of the focus. All the teaching models are meant for achieving some specific goals or objectives of teaching in relation to the environment of the learner. Therefore, objectives of teaching and aspects of the environment constitute the focus of the model.

Syntax

The term syntax or phasing of the model refers to description of the model in action. Each model consists of several phases and activities which have to be arranged in a specific sequence quite unique to a particular model. The syntax helps a teacher to use the model. It tells him how he should begin and proceed further.

Principle of reaction

While using a model how should a teacher regard and respond to the activities of the student is the theme of principle of reaction. These response should be quite appropriate and selective.

The social system

Models differ from each other with regard to the description of

(a) interactive roles and relationships between the teacher and student

(b) the kinds of norms which are encouraged and student behaviour which is rewarded. In some models, the teacher is the centre of activity, or activities. In some other model activities are equally distributed between teacher and students, while in others the students (a few or the whole group) occupy the central place. The leadership role of the teacher, the amount of control and the way in which the student behaviour is rewarded differs from model to model.

The support system

This element of model refers to the additional requirements beyond the usual human skills or capacities from the teacher and the facilities or schedules available in an ordinary classroom. They include films, self instructional system, visit to some place, etc. This generates a desirable classroom environment.

Application context

Some models are meant for short lessons, some for large and some for both. They also differ in terms of the goal achievements-conative, cognitive or affective. So each model through its element of application context tries to describe the feasibility of its use in varying context achieving specific educational goals and demanding specific work environment.

Families of Models

(1) Information Processing Models

This family of models aims at fostering, information processing ability of the learners. Information processing involves intellectual skills required to analyse information which include the ability to make observation, and through the use of inference, to generalize, to predict and to explain events. The models included in this family are:

Inductive Thinking model

Inquiry Training model

Scientific Enquiry model

Concept Attainment model

Cognitive Growth model

Advance Organiser model

Memory Organiser model

(2) Personal Models.

The models of this family share an orientation towards the individual and the development of self. They are more concerned with human feelings and emotions. The models included in this family are :

(1) Non-Directive Teaching

(2) Awareness Training

(3) Synectics

(4) Conceptual Systems

(5) Classroom Meeting

(3) Social Interaction Models

This family of models emphasizes the relationship of the individual to society or to other persons. Priority is given to the development of social skills, which help individuals to engage in a democratic process and to work productively in the society. The models included are

(1) Group Investigation
(2) Social Enquiry
(3) Laboratory Method
(4) Jurisprudential
(5) Role Playing
(6) Social Simulation.

(4) Behavioural Models

In these models the emphasis is on changing the observable behaviour of the learner rather than the underlying psychological structure and the unobservable bahaviour. The models in this family are

(1) Contingency Management
(2) Self Control
(3) Relaxation
(4) Stress Reduction
(5) Assertive Training
(6) Desensitisation
(7) Direct Training

Glaser's Basic Teaching Model

The Basic Teaching Model was developed by Robert Glaser on the basis of psychological principles. It is termed as 'basic' because it tries to explain the whole teaching process in an appropriate way dividing it into four basic components.

A. Instructional Objectives
B. Entering Behaviour
C. Instructional Procedures
D. Performance Assessment

Instructional Objectives

They indicate the stipulated goals that a student is supposed to attain upon completion of a part of instruction.

Entering Behaviour:

It is the initial behaviour of the student before the beginning of instruction. It is the basic potential or level of performance in terms of educational abilities comprising of the factors like previous knowledge of the subject, intellectual ability, motivational state etc. The assessment of the entering behaviour is a very significant aspect of an instructional process. Usually it is the starting point but in this model it occupies the second place. In actual teaching - learning situation both of them interact to influence and help each other for the success of the whole instructional process.

Instructional procedure

It is the most active and functional part of the teaching process. It represents the teaching methods, strategies and student teacher interaction patterns involved in the task of teaching. The stipulated instructional objectives and the entering behaviour work as a deciding base for the selection and use of instructional procedures.

Performance Assessment

Performance assessment is related with the task of assessing the performance of the learner. In the light of ones entry behaviour and stipulated objectives his terminal behaviour is assessed through some evaluation techniques like test, observation etc. The assessment of the performance prove as an effective 'feed back' device based on which objectives may be modified and instructional procedures may be improved.

Glaser's Basic Model description in terms of elements

Focus

The model tries to pinpoint the four basic function, processes and major activities comprising the whole teaching - learning process. The sequence to be followed in the instructional process is also high-lighted.

Syntax

The flow of activities in this model is sequential. First of all objectives to be achieved through the instruction are fixed. Attempts are made to assess the potentialities of the learners in terms of their entry behaviour. Then in the light of the entry behaviour the school instructional work is carried out to achieve the stipulated objectives and how far these objectives have been realized is ascertained in the last phase.

Principle of reaction

The main principles of reaction are

(1) Principle of active involvement and expertise

The teacher has to remain active. He has to acquire essential skills in the formulation of objectives, assessment of entry and terminal behaviour and devising suitable means and ways for the realization of set objectives. At every stage he has to develop proper understanding of the potentials and difficulties of his students to reach the goals.

(2) Principle of interdependence

The four stages involving objectives, entry behaviour, instructional process and assessment of its outcomes are quite interconnected and interdependent. The student's responses are to be understood and dealt within the light of such interaction and dependence.

(3) Principle of correction and follow up.

If the assessment in terms of terminal behaviour does not match with the aspiration in terms of the set objectives and entry behaviour then necessary activities for the improvement of instructional process should be carried out.

Social System

This model is structured to be dominated by the active role and control of the teacher and success depends upon the competency and ability of the teacher in terms of the acquisition of various skills like formulation of objectives, employment of methods, strategies of evaluation etc.

Support System

(i) Requirement of sufficient pre-service and in-service training facilities for teachers to acquire needed skills and competencies.

(ii) Need for the desirable teaching - learning situations and environment for the employment of suitable teaching strategies and instructional technology.

(iii) Need for appropriate evaluation devices for the assignment of entry as well as terminal behaviour of the pupils.

Applicability of the model

The model is applicable to any teaching - learning situation preferably dominated by the teacher and requiring the flow of knowledge and information in a quite systematic and structured way and realizing some well defined instructional objectives within the limited means of the usual classroom situation.

Concept Attainment Model

The content a teacher transact in a class room can be broadly classified into three categories, namely facts, concepts and generalizations.

Concept of a thing is what it means to an individual or a mental image of the thing formed by a generalization from particulars. Triangle, prime numbers are all concepts. Bruner and his associates asserted that we encounter innumerable stimuli in our environment and we respond to them in terms of this class membership rather than their uniqueness. Or other words we form categories of objects or events on the basis of their common characteristics and these categories are known as concepts.

Facts are singular occurrences which either occurred in the past or exist in the present and which have no predictive validity. Eg. India got independence in 1947.

A generalization is an inferential statement which expresses a relationship of two or more concepts, applies to more than one event and has a predictive and explanatory value. Eg. When a transversal cuts two parallel lines, the alternate angles are equal.

According to Bruner, a concept has five elements:

- Name of the concept
- Examples (positive and negative)
- Attributes (Essential and Non essential)
- Attribute values
- Rules

Understanding a concept means knowing all the elements of the concept. The name is the term given to a category or concept. The category of rectilinear figures

bounded by four line segments is called as quadrilateral and hence the term quadrilateral refers to the name of the concept/category. Items which are grouped in a single category may differ from one another in certain aspects but they will have certain common features and it is because of these common features they are given the one general name.

Examples refer to the instances of the concept. Some are positive and some are negative. Positive instances demonstrate what the concept is whereas negative instances demonstrate what the concept is not.

Each example can be described in terms of its basic characteristic called attributes. An attribute is any discrete feature of an event. Attribute of concepts may have a range of values. Eg. The attribute of 'Quadrilateral' may be represented by the values square, rectangle, parallelogram etc. There are concepts whose attributes do not have a range of values. Eg. If the concept is 'apple' then each fruit is an example. Mango , orange are negative and apples are positive examples. The colour may be an attribute and yellow or red may be attribute values.

A rule is a definition or statement specifying the essential attributes of a concept. Eg. polygon is a concept. The definition of polygon is the rule. A rule normally evolves at the end of the concept attainment process.

Bruner and his associates have done substantial work in determining the process of learning concepts. Based on this Joyce and Weil (1980) developed a model which is known as Concept Attainment Model.

In concept attainment teaching situation the learner is presented with an array of instances or examples that are alike in some ways, and different in others. The learner encounters with these examples and must find out whether each instance exemplifies the concept. Each instance or example provides potential information about the character and attribute value of the concept. The process of sorting 'Yes' and 'No' instance is the core of the concept attainment model of teaching.

Bruner and his associates distinguished between the two learning conditions of selection and reception. In selection, the examples are not marked Yes or No. The learner counters with the array of unmarked examples selects one and inquires whether it is 'Yes' or 'No'. In reception condition, the teacher presents the examples in a pre arranged order labeling them 'Yes' or 'No'.

Since people use different strategies to attain concepts, concept attainment model has three variation.

A. Reception Model

B. Selection Model

C. Unorganised Material Model.

(A) The Reception Model of Concept Attainment

Focus

The concept attainment model facilitates the type of learning referred to as conceptual learning, in contrast with the rote learning of factual information. The model works as an inductive model designed to teach concept through the use of examples. In addition to the attainment of a particular concept, the model also enables them to become aware of the process of conceptualizing.

Syntax

Phase one : Presentation of data and identification of the concept

(i) Presenting examples with 'Yes' or 'No' labels in a pre-arranged order by the teacher.

(ii) Comparing attributes in positive and negative examples.

(iii) Generating and testing hypotheses.

(iv) Naming the concept.

(v) Stating the rule or definition of the concept according to its essential attributes.

Phase Two : Testing attainment of the concept

(i) Correctly identifying additional unlabelled examples of the concept as 'Yes' and 'No'

(ii) Generating own example.

Phase Three : Analysis of thinking strategies

(i) Describing thought.

(ii) Discussing the role of hypothesis and attributes.

(iii) Discussing type and number of hypothesis.

(iv) Evaluating the strategies.

Principle of Reaction

(i) The teacher has to remain supportive of the students' hypothesis.

(ii) He has to maintain record by keeping track of the hypothesis of the attributes as they are mentioned by the students.

(iii) He has to remain supportive for turning the students' attention towards analysis of their concepts and strategies.

(iv) He has to encourage analysis of the merits of various strategies and attempt to seek the best strategy for all people in all situation.

The social system

In the initial phase of concept attainment model it is helpful to be very structured. In most part of the teaching, teacher has to exercise control over the social system. He has to present examples in such a way that the attributes are clear and there are positive and negative examples. Freedom should be given to the students for carrying out their own thinking. It they fail to reject wrong hypotheses, the teacher prompts them to re-examine the data.

The support system

(i) It requires materials in the form of well thought examples marked as positive and negative.

(ii) As the students have to describe the characteristics (attributes) of the examples (marked as 'Yes' or 'No') a black board is required.

(iii) A flannel board can be used to present the definition and explanation of the concept.

Application context

Concept attainment model proves an excellent way (based on inductive reasoning and systematic thinking) to teach the concept through the use of examples. It helps the students to acquire concepts which they are unfamiliar with. It can be used with students of all ages. Reception model is very appropriate for young children whereas the selection and unorganized strategies for secondary grades. The model is an excellent tool for evaluation.

B. The Selection Oriented Model

In the selection oriented model the teacher presents an array of examples. Examples are not labeled as 'Yes' or 'No'. A learner confronted with the examples, selects one and enquire whether it is 'Yes' or 'No'. The students may be asked for their own examples in order to attain the concept. The selection model places responsibility in the hands of the students.

Difference in the Syntax of Reception and Selection Models

(i) The reception model is highly structured and controlled by the teacher. The selection model permits more autonomy to the students.

(ii) Initially only two examples are provided in reception model. Whereas in the selection model all examples are displayed to the students from the beginning of the activity.

(iii) In reception model all the examples are labeled, the selection model requires unlabelled examples.

(iv) In reception model, the sequence of examples presented is decided by the teacher whereas in selection model, students enjoy the autonomy in controlling the sequence of the examples.

(v) The students may put their own examples in selection model and ask the teacher to respond in 'Yes' or 'No' in order to attain the concept.

In this model, learners can work individually as well as in groups. It is desirable that the learner works initially as individuals and later in groups.

C. The Model of Unorganised Material

It is similar to Reception and Selection Models in the following ways.

(i) It has a common target of helping the students to attain concept.

(ii) It also uses examples to teach a concept.

(iii) It also begins with the identification of one positive and one negative example of the concept to be attained.

It is different from the other two models in the following ways.

(i) Leaving the first two, all other examples are provided by the students.

(ii) It provides more autonomy to the students.

(iii) The data presented is quite unorganized and unstructured.

Inquiry Training Model

Whenever we are engaged in the process of 'finding out' or 'investigating' through questions, we are involved in the process of 'inquiry'. If the inquiry takes the form of disciplined and systematic approach, it becomes the spirit of scientific method. Inquiry Training Model was developed by Richard Suchman to teach students a process for investigating and explaining unusual phenomena.

Focus

The children are very curious by nature. For satisfying their curiosity they may be seen engaged in the inquiry process, exploring and analysing the things or phenomena by themselves. The Inquiry Training Model aims to refine or improve such inquiry skills of the children through a systematic inquiry training.

Inquiry training begins by presenting the students with a puzzling event. This event should be puzzling or mysterious enough so that its solution is not straight forward or obvious, only then would the event arouse the curiosity. Suchman called such an event 'discrepant event' as this event would not be in tally with the ideas or concepts already familiar to the learners, causing intellectual confrontation. Such confrontation would lead to the discovery of new knowledge.

It provides systematic structure within which the students have to ask questions, organize them to formulate and test hypotheses in the similar way as scholars and scientists do in organizing knowledge and generate principles for explaining causation.

Syntax

Inquiry Training Model consists of five phases - encounter with problem, the three stages of inquiry (verification, experimentation and explanation), followed by the analysis of the inquiry.

Phase One

Encounter with the problem

(i) Explaining Inquiry procedures.

(ii) Presentation of the problem or puzzling event.

Phase Two

Data Gathering Process (verification)

(i) Verifying the nature of objects and condition.

(ii) Verifying the occurance of the problem.

Phase Three

Data Gathering Process (Experimentation)

(i) Isolating relevant variables.

(ii) Hypothesizing (and testing) causal relationships.

Phase Four

(i) Formulating an explanation.

(ii) Formulating rules or explanation.

Phase Five

(i) Analysis of the Inquiry Process

(ii) Analysing inquiry strategy and developing more effective ones.

The first phase is the student's confrontation with the puzzling situation. The teacher presents the pre-planned discrepant event and explain the enqiry procedure, the objectives and direct the students to ask questions. The question must be answered by a 'Yes' or 'No'. Students may not ask the teacher to explain the phenomena to them.

Phases two and three belong to the data gathering operation of verification and experimentation. Data is gathered by the students in two ways.

(i) by asking questions to be responded by the teacher in 'Yes' or 'No' form.

(i) to conduct a series of experiments on the environment of the problem situation.

The process of verification is carried out in two steps.

(i) verifying the nature of objects and events like

(a) identify the objects

(b) state or behaviour of the objects at a particular time.

(c) nature of an action or happening.

(ii) verifying the occurance of events.

This phase requires the teacher to be aware of the type of questions to verify all aspects of the problem and change questioning pattern through intervention.

The experimental phase is directed at exploration and direct testing. Hypotheses involving plausible explanation are formulated and tested by the students. The teacher has to remain very careful in guiding and directing the students for the rejection of the inappropriate hypothesis.

In phase four the teacher calls on the students to formulate an explanation. It is useful to ask many students to state their explanations so that the range of differences is revealed. Together the group can shape the explanation that fully responds to the problem situation.

Phase five requires working of the teacher and the students together for analyzing one another's strategies. They can evaluate the modes of questioning and responding, the appropriateness of questions, the information they needed but did not obtain, the suitability of the formulated hypothesis and its verification may also be properly discussed and evaluated. This analysis helps the teacher to modify and improve the inquiry process and equip the students with appropriate problem solving skills.

Principle of Reaction

Rules for responding and reacting to the actions of the students are:

(i) Insuring that questions are phrased so they can be answered as 'Yes' or 'No'.

(ii) Asking students to rephrase invalid question.

(iii) Pointing out unvalidated statements. Eg. We have not yet established that this is a polynomial.

(iv) Using the language of the inquiry process - identifying student questions as theories and inviting testing.

(v) Neither approving nor rejecting student theories.

(vi) Pressing students for clearer statements of theories and more support for generalizations.

(vii) Encouraging interaction among students.

Social System

Inquiry Training Model provides high weightage to the controlling of social system. Teachers and students participate as equals where exchange of ideas is concerned. At every stage teacher has to respond in such a way that students may be encouraged to initiate and persuade the inquiry as much as possible. In the beginning, the social system may be highly structured, it may be relaxed when the students seem to learn the principles of inquiry. The open environment with a possibility of proper teacher - pupil interaction and pupil-pupil interaction surely adds to the success of this model.

Support System

A teacher requires additional support in the form of

(i) a set of confronting matérials.

(ii) technical understanding of the intellectual processes and strategies of inquiry.

(iii) resource material bearing on the problem.

Application Context

Basically the Inquiry Training Model has been developed to provide training in systematic inquiry. It is not a method of teaching or imparting information. But it was found useful in natural sciences. Any topic or event from the curriculum area which can be converted into a problem situation or puzzle can be selected for inquiry training.

Advance Organiser Model

This model is based on the learning theory formulated by David.P.Ausubel. Although modern educationalists condemn method of teacher presentation in the class, Ausubel thought by improving the technique of presentation meaningful learning takes place. He stresses mastery of academic material as an important goal of classroom teaching. What is required is to make it integratged to the existing cognitive structure.

Ausubel introduced Meaningful Verbal Learing and says that verbal learning becomes inferior only when it degenerates into rote learning instead of meaningful learning. His theory of meaningful verbal learning deals with three concerns.

(1) how knowledge is organized.

(2) how mind works to process new information.

(3) how teachers can apply these ideas about curriculum and learning when they present new materials to students.

The key concepts in meaningful verbal learning are:

a. Student's existing cognitive structure

Cognitive structure refers to a student's knowledge of that matter with special reference to how much he knows, how well he knows and how effectively the knowledge is structured. How meaningfully a student can learn new material depends on his cognitive structure. Hence before the new knowledge is presented the teacher should ensure the clarity and sufficiency of the existing cognitive structure. It is in the absence of this that verbal learning degenerates into rote learning.

b. Meaningful learning set.

This is a condition in which connection could easily be established between the new material to be learnt and the related materials that already exist in the cognitive structure. Such a mental set makes the learner ready to receive the new knowledge.

c. Structure of concepts.

Each discipline has a structure of concepts, hierarchically organized - broad concepts at the top and more concrete concepts at the lower stages. While organizing knowledge in the cognitive structure, human mind has a tendency to follow the same hierarchical order. The sequencing of concepts (idea) is of utmost importance in meaningful verbal learning.

d. Advance Organizer

These are so called because they form the preliminary means of organizing and strengthening cognitive structure. These ensure meaningful reception and effective retention. These are materials presented in advance as introduction to the new materials to be presented. The advance organizer will include items of the cognitive structure of the learner with which the new material can be linked when they are later presented. This can include terms, facts, concepts already familiar to the learner. Though an organizer will be familiar to the learner, it needs further clarification and a reteaching to ensure that the new items are meaningfully linked.

There are two types of advance organizers

They are 1. Expository

2. Comparative

Expository organizers are those in which a general class (group) relationship is exposed with a view to include more specific classes Eg. Concept of pull or push and force can be used as an organizer for teaching specific types of forces, laws of motion.

Comparative organizers are those that contain concepts similar to the ones to be presented so that familiar relations are established and learning made meaningful. This may be in the form of analogies which would help to establish meaningful linkages.

e. Subsumer

The advance organizer acts as a subsumer, ie, a structure which can subsume (contain or include) all the new materials presented. The linkage established with the existing familiar items of the subsumer helps the learner in acquiring meaningful learning. It serves the function of a strong 'platform' on which the new knowledge can be fixed as part of the cognitive structure.

f. Progressive Differentiation

Ausubel thinks that the hierarchical order in the processing of information is from the broader and more general to the specific ones. It is the process of maintaining this gradation by which specific items are gradually presented one by one in the hierarchical order that is known as Progressive differentiation.

Eg. If the general concept of a quadrilateral is systematically integrated in the existing cognitive structure, the specific items such as trapezium, parallelogram, rhombus can be presented one by one in comparison with the existing subsumer, which is the thorough understanding about quadrilateral.

g. Integrative reconciliation

This is the term that explains the conscious effort to relate new items to previously learnt content in such a way as to make the cognitive structure an integrated whole. This naturally follows progressive differentiation if it is systematically done in relation to the advance organizer. This ensures the development of a well integrated cognitive structure.

The pedagogic strategies (steps in learning a new lesson) leading to a meaningful verbal learning were deducted on the basis of key concepts. These steps include:

1. Systematic organization of the new learning material.
2. Identification of the terms essentially required in the learner's cognitive structure for meaningful learning of new material.
3. Updating the existing cognitive structure as per the requirement by filling up the gaps if any and bringing the essential items to the conscious level. This is done in the form of presentation of the advance organizer.

4. Presenting the new material in proper sequence from the more inclusive general items to the specific ones.
5. Helping the learner to struggle with the learning material and enabling him to connect each to the essential pre-requisite in the updated existing cognitive structure, the advance organizer.
6. Making the learner register and integrate the new learning material as part of his cognitive structure.

Ausubel says that by adopting such a systematic style of presentation the teacher can help the learner to meaningfully master the maximum information in the least time.

Description of the Model

Syntax

The Advance Organiser Model has three phases of activity.

Phase one is presentation of the advance organizer. This phase consists of three activities : clarifying the aims of the lesson, presenting the advance organizer and prompting awareness of relevant knowledge.

The teacher should ensure that the existing cognitive structure contains all the essential pre-requisites for meaningfully receiving and linking the new learning materials. This can be ascertained by discussion, questioning and feed back. If deficiencies or gaps are noticed those should be filled at the time of presenting the advance organizer.

Phase two is the presentation of the learning task or material. In this phase an important task is to be maintain students' attention. When once the efficiency of the

existing cognitive structure is ensured present the minor concepts in the order using the principle of progressive differentiation. A variety of tools and techniques like pictures, aids, films, examples, analogies, action could be helpful for making learning meaningful. Discussions and arguments can also be used.

Phase three is to anchor the new learning material in the students' existing cognitive structure. There are four activities in this phase.

- Promoting integrative reconciliation.
- Promoting active reception learning
- Eliciting a principle approach to subject matter.
- Clarification.

Social System

Teacher has the control of the intellectual structure. During the first two phases it is highly structured but during the third phase more free interaction occurs.

Principle of reaction

The teacher is seen as presenter of the learning material. The teacher's reaction will be guided by the purpose of clarifying the meaning of the new learning material differentiating it from and reconciling it with existing knowledge, making it personally relevant to the student and helping to promote a critical approach to knowledge.

Support System

Well organized learning material that includes the advance organizer and the new items to be successively differentiated form the most important support. Instructional materials can be prepared in advance.

Applicability of the Model

The Advance Organiser Model is designed for use in face-to-face teaching in the form of lectures and explanation. This model can be used in developing instructional material. This model has great potential in teaching concepts, relationships and imparting information effectively. It can be used to teach any subject. This model is very effective for high school and higher secondary school students.■

CHAPTER - 11

RECREATIONAL MATHEMATICS

According to Bertrand Russel "Mathematics is considered as the subject in which we never know what we are talking about, nor whether what we are saying is true". The impression about mathematics as a dry subject can be reversed with the help of recreational activities in mathematics.

In ancient times, mathematics was mainly studied for its recreational value and as a leisure time activity. Playing with numbers and mathematical problems was enjoyed by people. W.F.White observed "amusement is one of the fields of applied mathematics".

Importance of Recreational Mathematics

- Recreational activities bring a healthy change in the classroom atmosphere.
- It develops a taste for mathematics.
- The students learn to appreciate the power and beauty of mathematics.

- It helps in developing much abstract relationship which would otherwise remain vague.
- It sharpens wit and stimulates quick thinking.

Organising recreational activities in Mathematics

For deriving maximum possible benefit, recreational activities must be organized in a proper way.

- The teacher should fix a period in timetable for recreational activities.
- An active student should be given the responsibility for organizing the activities.
- The teacher should see that all the students are actively involved in the recreational work.
- Overuse of riddles should be discouraged.

Magic squares, Riddles, Puzzles, Number patterns, Paradoxes, Unsolved problems.

Riddles

A riddle can be defined as an interrogatory statement that has two meanings and is generally asked as a puzzle to be solved by the observant. Most of the riddles have a clue within themselves and it lies in the creativity and presence of mind of the observer to crack it.

Riddles are actually ancient games and archaeologists have found evidence of riddles that date back thousands of years. According to the evidence short riddles were used as part of religious and mystical rites.

Example- How do you know that the following fractions are in Europe.

$\frac{A}{C}$ $\frac{X}{C}$ and $\frac{W}{C}$

Ans. : Their numerators are all over C's.

Number Patterns

A number pattern is a series of numbers that follow a rule.

Example Fibonacci Sequence

0,1,1, 2,3,5,8,13,21,34,55,89,144 ....

Each number in the sequence is the sum of the two preceeding numbers starting with the root number 1. The Fibonacci numbers are nature's number system. They appear everywhere in nature, from the leaf arrangement in plants to the pattern of the florets in a flower.

Puzzles

Puzzles are problems intentionally designed to stimulate brain. Puzzles have been ingrained into the human culture since the very beginning. One of the oldest and most popular puzzles in Tangram, which has been testing minds all over the world for thousands of years. The extent to which puzzles have become part of daily life can be ganged from the popularity to crosswords and Su-do-ku.

Paradox

Paradox is anything which often appears to be false, but is actually true or which appears to be true, but is actually false or which is simply self contradictory. It is a statement or group of statements that leads to a contradiction or a situation which defies intuition.

Example:

In the centre of a circle 8ft. radius is a frog. It begins to jump in a straight line to the circumference of the circle. Its first jump is 4ft., its second 2ft. and it continues to jump half the length of the proceeding jump. How many jumps does the frog make to get out of the circle.

Magic Squares

A magic square of order 'n' is a rectangular array of n^2 numbers, usually distinct integers, in a square, such that the n numbers in all rows, columns and both diagonals sum to the same constant. The first magic square in history was created in China by an unknown mathematician, probably sometimes before the first century A.D called Lo Shu Square. A well known early 4 x 4 magic square in India can be seen in Khajurao. It dates from the 10^{th} century.

Unsolved Problems

A problem which cannot be solved is known as unsolved problem. Mathematical puzzles vary from the simple to deep problems which are still unsolved. The whole history of mathematics is interwoven with mathematical games which have led to the study of many areas of mathematics. Unsolved problems are challenging and they develop curiosity among students.

Examples: Kirkman's School Girl problem.

The problem, posed in 1850, asks how 15 school girls can walk in 5 rows of 3 each of 7 days so that no girl walks with any other girl in the same triplet more than once.■

CHAPTER - 12

THE PROFESSIONAL PREPARATION OF TEACHERS OF MATHEMATICS

In the educational crisis that has arisen during recent years, mathematics, as an integral part of the curriculum, has found the voice of criticism rather severe. The justification for such vehemence is not in the short comings of the subject, one which through the ages has been a beacon light to scientific discovery and intellectual progress, rather such justification is to be found in the poor instruction imparted by unprepared and non-enthusiastic teachers. It is poor teaching that leaves the impression that mathematics is merely a tool subject composed of a mass of signs, symbols, and laws of operation. There is a great need for the studious definition of a program designed to prepare prospective teachers for a true profession of teaching of mathematics.

There are two equally important aspects of any true profession, viz, significant knowledge and effective technique. One cannot be efficiently professional if there is any distinct weakness in either aspect. A true functional program of professional preparation must therefore place emphasis on acquiring knowledge significant to the chosen profession and also on the acquaintance with and use of the more efficient techniques of that profession.

There are teachers who know things well, but because of their lack of patience with student difficulties and their willingness to adjust their teaching to the varying abilities of students, actually destroy the interest of many students in mathematics when, under more favourable conditions, it might have been made to flourish. Such teachers lack that professional attitude which should impact them to discard any sense of intellectual superiority and to view the subject through the eyes of the immature student so that they might patiently guide and encourage him in his efforts and stimulate his interest in further exploration of the field of mathematics.

Masterful Scholarship in a body of relevant knowledge is an absolutely essential for effective teaching, but it must be supplemented by a proficiency in the use of efficient techniques of instruction. None should be emphasized to the exclusion of the other, but a proper balance should be maintained through out the preparation program.

Significant Knowledge

A professional attitude should be a *sine qua non* for every teacher of mathematics. The term "Professional attitude" is interpreted here to mean an enthusiastic interest in mathematics as a chosen field of study and service, an inspired concept of the value of mathematics in the structure of civilization, and an eager readiness to interpret carefully and thoughtfully those fundamental laws, mechanical processes, generalizing procedures, and possibilities of practical applications which so definitely

characterize mathematics as a field of study and endeavour. In addition to providing a general education for cultural background, the program for the professional preparation of teachers of mathematics should equip such a teacher with an integrated philosophy of education, a devotion to teaching as a profession, and a sense of responsibility for the contributions he will be expected to make in his chosen field of work. This body of professional knowledge should be provided through courses designed to acquaint the individual with the place and function of education in our social order, the inter relationships that exists between the various professions, and the manifold opportunities for service which present themselves to teachers; to build up 'sympathetic understanding of the mental, physical, and social characteristics of the children or adults to be taught'; and to provide 'opportunities for acquiring a 'safety minimum of teaching skill' through observation, participation, and actual practice under supervision'.

The teacher of mathematics is the seller of mathematics. It is he who must convince the consumer of the value of his subject and, through the medium of efficient service, secure and retain consumers. He is constantly confronted with questions as to the value of mathematics as an asset to the individual and as a significant element in the program of general education. His interest in mathematics must embrace its educational implications and practical applications as well as its intrinsic subject appeal. Thus the preparation of the teacher of mathematics should emphasize that type of scholarship which seeks to integrate the subject with broad fields of learning and to relate it to general human activity and interest. The teacher must learn to evaluate mathematics in the light of its role in the history of civilization, its contribution to the present social order, and its relation to future progress.

It is the responsibility of the teacher to assist immature learners in the mastery of mathematics. The teacher of mathematics should not only strive for proficient mastery of the subject, but he should also make every effort to be conscious of the process by which he arrives at that mastery. He should pause at significant points for moments of reflection in which he should attempt to analyze the learning process involved and to evaluate the materials studied.

Relevant Scholarship

The teacher of mathematics should have some appreciation of the part that mathematics has played throughout the centuries of progress. Further, he should have those contacts with the subject matter and history of mathematics that would enable him to formulate an intelligent notion of the meanings of mathematics. In 1901 Bertrand Russell defined mathematics in words that are superficially facetious but fundamentally significant when he said : "Mathematics may be defined as the subject in which we never know what we are talking about, nor whether what we are saying in true".

The competent teacher of mathematics should realize that in any system of constructive thought the validity and consistency of the assumptions and definitions upon which the conclusions are based. It is important that students should have this points of view, and the teacher should make every reasonable effort to assist them in acquiring it. From a logical point of view the set of fundamental assumption should be consistent, independent, and categorical, but pedagogically the only essential requirements are that they be consistent and categorical. An appreciation of this dependence upon fundamental assumptions and definitions and previously established theorems should help to develop an "if-then" mental attitude which should function in a more intelligent interpretation of human events.

Significant Professional Techniques.

The demands made on the teacher of mathematics in the modern program of education make it absolutely essential that he knows the orientation of his field of work in the entire secondary program; that he be familiar with significant objectives and problems in mathematics; that he knows the techniques of selection of text books, work books, and other teaching equipment; that he be familiar with the fundamental philosophy of significant evaluation of instruction; that he be skilled in the construction use and interpretation of tests: factual and functional, standardized and non-standardized, objective and essay; that he be acquainted with various instructional techniques and know when and how to use them for maximum efficiency; and that he be prepared to assume his share of the responsibility in the pupil guidance program.

The teacher of mathematics must be thoroughly familiar with the complementary problems of transfer of training and individual differences. He must know the major sources of student difficulties, how to diagnose these difficulties and plan programs of remedial teaching; he must know how to plan an instructional program and how to adapt this program to different ability groups; he must be enthusiastically interested in mathematics and know the fundamental principles of the psychology of motivation; and he should know how to detect and remedy for inefficient study habits and techniques. Finally, the teacher of mathematics should be well versed in the fundamentals of the psychology of learning and in their application of materials and methods for better instruction in mathematics.

The teachers should be equipped to read, interpret, and evaluate the published results of experimental investigation and to make use of significant findings in the improvement of their own teaching procedures. It is also very desirable that they be equipped to pursue scientific investigations in connection with their own program and to interpret intelligently their findings for the benefit of others.

The future of mathematics in schools is primarily the responsibility of the teacher of mathematics in these schools. Its status will depend largely upon his ability to present and interpret his subject as a worth-while educational venture. The professional preparation of this teacher should equip him with the scholarship and techniques essential to the satisfactory fulfillment of his professional obligation. He must be able to organize and present mathematics in such a way that the children will be brought not only to a realization of the intrinsic nature and value of mathematics itself but also to an equally clear realization of its role in enabling man to relate, understand, and control his environmental factors and to direct his social and economic advancement.

If these ends are to be attained, the training of the teacher must be a continuous process. It should make the teacher unwilling to permit himself to stagnate under a comfortable self-complacency. It should inspire him to incessant effort both in the expanding of his mathematical horizons and in educational experimentation directed toward the improvement of his instructional techniques. The teacher so prepared will not restrict his attention and his instruction to the confines of mere operational mechanics. He will lead his students with enthusiasm into realms of mathematical thought

and endeavour which will be both stimulating to their curiosity and intellectual interest and broadly significant to their insights, appreciations and general cultural development. ■

CHAPTER - 13

QUANTIFICATION OF DATA

For the purpose of evaluation, the data has to be expressed quantitatively in the form of marks or scores. It is on the basis of these scores that judgments are passed regarding the performance of students. This is possible only if the scores are analysed and meaningfully interpreted.

Graphical representation of quantified data

Classified data can be represented by a variety of graphs with a view to give a visual perception of the distributions.

Construction of graphs

There are different types of graphs representing the relation between two variables, but the procedure involved in their construction has certain aspects in common. A graph represents the relation between two variables, so it is two dimensional. Each of the two variables namely X and Y is represented on an axis. The two axes are perpendicular lines, one drawn horizontally and the other perpendicular to it on the left side. These are the X-axis and Y-axis respectively. The intersection of the axes, is the origin. In a frequency table, the scores are marked on the X-axis and the frequencies on the Y-axis. After marking the axis, a convenient scale for marking the variables has to be found out.

Frequently used graphs are frequency polygon, smooth curve, and histogram.

Frequency Polygon

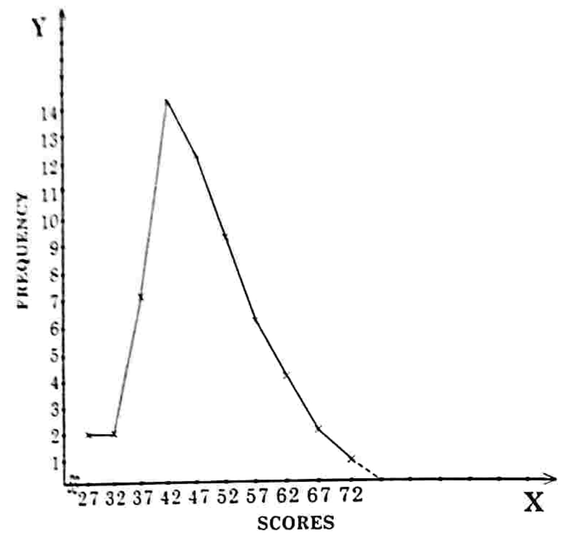
In marking the variables on the axes, the scores are to be first marked on the X-axis. In frequency polygon, it is the midpoint that represents a score. The frequencies are marked on the Y-axis. Corresponding to each score and its frequency, the points are plotted on the graph. After plotting all points, these may be joined in the order by line segments. Such a graph is known as broken-line-graph. The ends of this graph are joined to the X-axis to make it complete. Then the graph together with the X-axis makes a many sided closed figure or a polygon. Since this polygon represents a frequency distribution, the graph is called frequency polygon.

An example is given below:

Scores	Frequency
70-74	2
65-69	3
60-64	4
55-59	6
50-54	9
45-49	11
40-44	14

Scores	Frequency
35-39	7
30-34	2
25-29	2

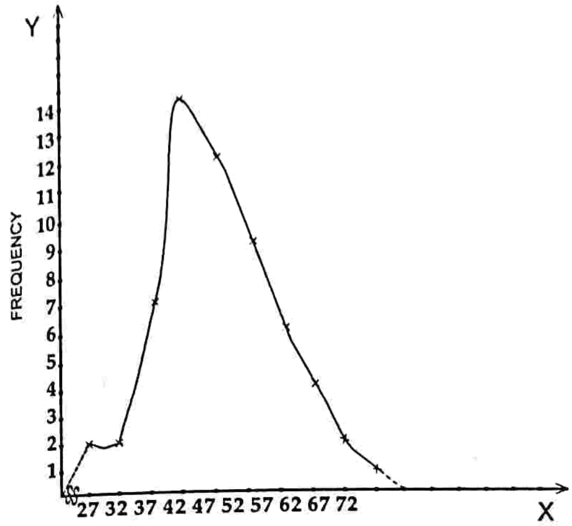
SCALE
X axis: 1 unit= 5 scores
Y axis: 1 unit= 1 frequency



Smooth Curve

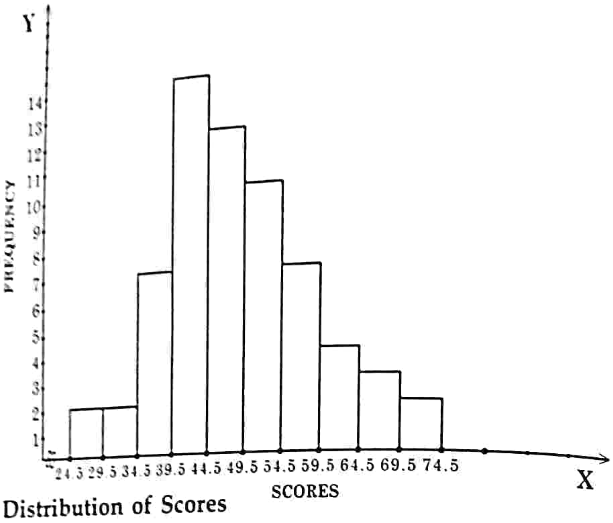
In the frequency polygon, the points are connected by line segments. In such cases, the intermediary changes are not considered. There may be cases where the changes of the Y-variable follow some pattern, showing continuity, so that the intermediary points also are significant. In such a case, the points of the graph are better connected by a free hand curve drawn smoothly. The smooth curve for the frequency distribu-

tion shown above will look like the one given below.



Histogram

In histogram the limits of each class are marked on the X-axis to represent the scores. Considering the frequency of a class to be distributed equally throughout the class, it is represented by a bar whose height is equal to the frequency, to the scale adopted. This is done by marking two points at a height equal to the frequency, above the two end points of the class and joining these to complete the bar. The bar conveys the idea that the frequency of the class as a whole is that much. When all the frequencies are thus represented by bars we get a diagram consisting of a number of bars, touching one another in the order. Such a graph is called histogram. The histogram for the data used for frequency polygon is given below.



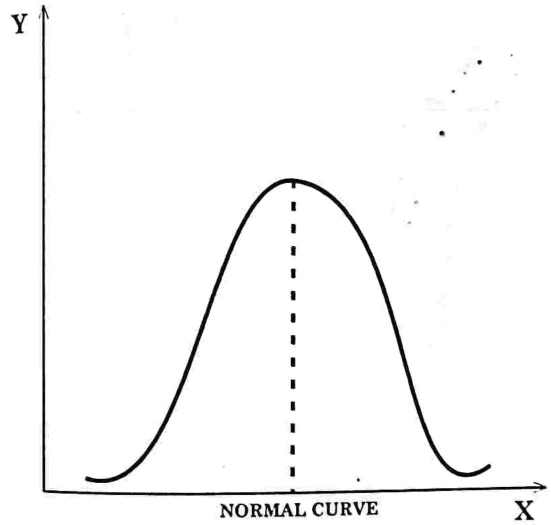
Distribution of Scores

Normal Distribution

If we examine the frequency tables a peculiar characteristic can be noted, regarding the distribution of scores. The frequency of the scores at both the ends is few in number, the majority of the cases concentrating in the middle. This characteristic is revealed by the graphs representing the frequency distribution also. The graphs have maximum height at the middle and at both ends the height gradually diminishes as indicated by its movement towards the X - axis. This indicates a phenomenon noticed in the distribution of many variables in nature. A very large number of endowments both physical and psychological are distributed among individuals in a predictable pattern. For example intelligence mea-

sured in terms of I.Q. If a very large sample of a population is selected and the I.Q of the members classified , it can be seen that those with very low and very high I.Q's will be comparatively small in number. As we move from both ends towards the middle, the frequencies of the group will gradually increases and when the middle group is reached the maximum frequency will be noticed. If the data of the distribution is represented by a smooth curve, it will be noticed that the maximum height of the curve will be exactly at the centre, the height gradually decreasing on either side, symmetrically, until the ends reach almost up to the X - axis. Because of this symmetry the graph will have the shape shown below, that is the shape of a bell.

This curve is known as normal probability curve, because it represents the probability of the nature of dis-



tribution of a particular phenomenon among the members of the given population. An exact symmetrical distribution is only theoretical. In practice, if the sample is large enough and randomly selected to represent a population, the smooth curve representing the distribution of the obtained scores will be very close to the normal probability curve.

In what ways can a curve representing a frequency distribution vary from the normal curve? Such variations are usually studied with respect to two phenomena, namely 'Skewness' and 'Kurtosis'.

Skewness

Fig(i) is a normal probability curve. In fig (ii), the curve extends more towards the right side of the maxi-

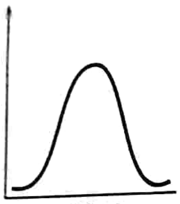


Fig (i)

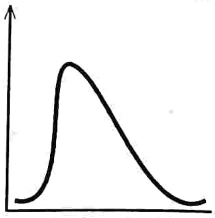


Fig (ii)

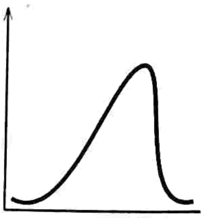


Fig (iii)

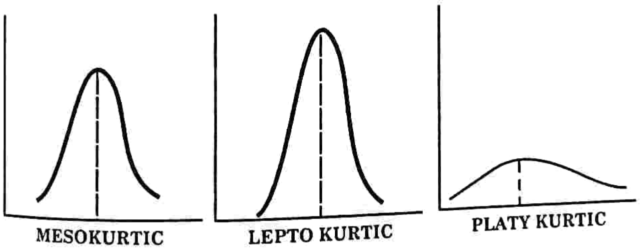
mum frequency, this extension being in the shape of a prolonged tail. In fig (iii) this extension is towards the left of the highest frequency. Hence it is clear that the second and third curves do not represent a normal distribution , as symmetry in the distribution of scores is lost. This lack of symmetry due to extended tails in a particular direction is known as 'Skewness'. If the tail extends to the right the distribution it is said to be positively skewed. If it is towards the left, the skewness is said to be negative. Positive skewness indicates clustering of frequencies more at lower scores whereas in negative skewness more scores are of high magnitude.

Many of the statistical interpretations of quantified data are made with the assumption that the scores are distributed normally. Hence before interpretation is attempted, a distribution has to be tested for its normality.

Kurtosis

Suppose a curve shows no skewness. There is no guarantee for its being normal.

All the above three curves posses symmetry but it is evident that the nature of distribution of the scores are



quite different. In fig (i) the distribution is fairly even with a gradual slope on either side. In fig (ii) the change in the frequency is very rapid when the scores go nearer to the centre, as indicated by the very high 'peakedness'. In fig (iii), the rise is very slow and hence not very marked at the centre. This has resulted in the peak being flat. The nature of the 'peakedness' is known by the name 'kurtosis'. The first curve shows an ideally normal distribution where as the other two distributions may be considered as 'abnormal'. The three curves may be described respectively as 'Meso Kurtic', 'Lepto kurtic', and 'Platy Kurtic'. ■

CHAPTER - 14

INTERPPRETATION OF QUANTIFIED DATA

Central Tendency

Teachers often pass judgments on their pupils. What is the basis for such comments? Though not very scientifically, they are making comparison of the children on the basis of some expected standard. In most cases this standard will be the average of the group which the individuals belong. When the average changes the nature of the judgment also changes. Why is that the average is taken as the basis while passing judgments? The characteristics of a normal distribution give an answer to this question. Natural endowments are distributed among the members of a large population in a specific pattern. Since the frequencies will be more in the middle than at the extremes, maximum number of cases will be clustering round the middle or the average of the group. In short the majority of a group always shows a tendency to be closer to the average. This phenomenon is said to be the 'Central Tendency'. The different measures of central tendency are the 'Arithmetic Mean', 'Median', and the 'Mode'.

Arithmetic Mean

Arithmetic Mean is nothing but the common average. It is calculated as the sum of the scores divided by the total number of cases.

The formula is $A = \frac{S}{N}$

where A represents average, S the sum of scores and N the total number of cases.

For example, consider the numbers 10, 12, 17, 8, 9,. 16, 20, 4.

$A = \frac{S}{N}$

S = 10+12+17+8+9+16+20+4 = 96

$A = \frac{96}{8} = 12$

When a single score is repeated often and a table showing the frequency is available, the formula needs some adjustment.

Example

Scores (x)	Frequency (f)	fx
20	4	80
30	0	0
40	2	80
50	5	250
60	3	180
70	1	70
	N = 15	660

Arithmetic Mean, $A = \frac{\sum fx}{N}$

where

$\sum$ - means ' the sum of'

f - frequency

x - score

N - sum of the frequencies.

$A = \frac{\sum fx}{N} = \frac{660}{15} = 44$

When the data is in the form of a frequency table in which scores are given in terms of classes, midpoint of a class may be considered as a single score and arithmetic mean is calculated with a slight modification in the formula.

Example

The table shows the distribution of marks in mathematics scored by 60 students in a test paper.

Scores	f	x Midpoint	fx
70 - 74	5	72	360
65 - 69	6	67	402
60 - 64	7	62	434
55 - 59	10	57	570
50 - 54	12	52	624
45 - 49	10	47	470
40 - 44	4	42	168
35 - 39	3	37	111
30 - 34	2	32	64
25 - 29	1	27	27
		N=60	$\sum fx = 3230$

Arithmetic Mean, $A = \frac{\sum fx}{N}$

$= \frac{3230}{60}$

$= 53.88$

When large numbers are involved, this type of calculation is laborious. Hence an easier method is used. This involves the following steps :

(i) Assume the midpoint of any convenient class as the arithmetic mean. This may be called the assumed mean and represented by the symbol 'M'. For the above example the midpoint of the class 50-54 that is 52 can be assumed as the mean..

(ii) Write down the deviations of the remaining classes from the assumed mean class, in terms of the number of classes by which each is above or below. This may be indicated by +1, +2, +3,...... for classes covering higher scores and -1, -2, -3,........ for the lower ones. The class deviation is represented with the symbol 'd'.

(iii) Multiply each of the above class deviations by the respective frequency and find out the sum.

(iv) calculate the arithmetic mean using the formula

$$A = M + \frac{\sum fd}{N} \times c$$

Where c is the width of the class.

Example:

Scores	f	d	fd
70 - 74	5	+4	+20
65 - 69	6	+3	+18
60 - 64	7	+2	+14
55 - 59	10	+1	+10
50 - 54	12	0	0
45 - 49	10	-1	-10
40 - 44	4	-2	-8
35 - 39	3	-3	-9
30 - 34	2	-4	-8
25 - 29	1	-5	-5
	N = 60		$\sum fd = 22$

$$A = M + \frac{\sum fd}{N} \times c$$

$M = 52$, $\sum fd = 22$, $N = 60$, $C = 5$

Arithmetic mean, A $= 52 + \frac{22}{60} \times 5$

$= 52 + 1.8 = 53.8$

Since the arithmetic mean takes into consideration the size of each score in a distribution, it is considered to be the most representative measure of central tendency.

The Median

Median is the middle score of a distribution. It is the score situated exactly in the middle when the scores are arranged in the ascending or descending order of their magnitude.

Example: Suppose five students scored the following marks in a test 45, 25, 30, 48, 52.

When arranged in ascending order it takes the form 25, 30, 45, 48, 52.

The middle score is 45. Hence it is the median.

If two scores are there in the middle median is the average of the two scores.

Example: 25, 30, 45, 48, 52, 56

$$\text{Median} = \frac{45+48}{2}$$

$$= \frac{93}{2}$$

$$= 46.5$$

Median for Frequency Distribution

The calculation of median for a frequency distribution involves the following steps.

(i) Determine the number of cases falling below or above the median. Since median is the middle score, half the number (N/2) falls above it and the remaining half below it.

(ii) Determine the class in which the median score can be expected. For this start from the lowest class and add up the frequencies one by one upwards, till the cumulative frequency is found to just exceed (N/2). The class at which this happens will contain the median. In other words, median may be the lower limit of the class and 'something'. This lower limit is represented as 'l'. The something to be added to 'l' to reach the median is known as the 'Score Distance'.

(iii) Determine the score distance as follows

Determine the cumulative frequency up to the median class, that is, the total frequency including the frequency of the class just below the median class. Let this cumulative frequency be 'm'. Now the additional frequency to be covered to reach the median will be equal to $\left(\frac{N}{2} - m\right)$

This has to be taken from the frequency of the median class (f). That is $\frac{\left(\frac{N}{2} - m\right)}{f}$ has to be added to l to reach the median. If the class interval is c, the score distance to be added to l becomes $\left(\frac{\frac{N}{2}-m}{f}\right)xc$.

(iv) Calculate the median using the formula

$$\text{Median} = l + \left(\frac{\frac{N}{2}-m}{f}\right)xc$$

Where l = lower limit of the median class

m = cumulative frequency up to the median class

f = frequency of the median class

N = total frequency

c = class interval.

The Mode

In a sufficiently large distribution certain scores or particular score may be seen to occur more frequently than others. The frequencies of such scores may be comparatively large. The score that has the maximum frequency in a distribution is said to be the model score or the mode.

For example, consider the set 20, 25, 18, 36, 25, 38, 15, 25, 40, 25. Among these 25 occurs maximum number of times. Hence it is the mode of the given set. In some cases, there may be two or even three scores with the same number of frequency. Such distributions are said to be 'bi model' or 'tri model'.

If the distribution is represented by a frequency table, the class with the maximum frequency may be

considered as the 'model class' and its midpoint as the mode. If l represents the lower limit of the model class, f_1 the frequency of the class just below the model class, f_2 the frequency of class just above the model class and c the class interval, mode can be calculated as

$$M_0 = 1 + \frac{Cxf2}{f1+f2}$$

For example

	Scores	Frequency
	70-74	5
	65-69	6
	60-64	7
	55-59	10
Model Class →	50-54	12
	45-49	10
	40-44	4
	35-39	3
	30-34	2
	25-29	1

$$M_0 = 1 + \frac{Cxf2}{f1+f2}$$

In the above table, the model class is 50-54, the frequency of the lower class is 10 and that of the upper is 10, so mode is

$$M_0 = 49.5 + \frac{5x10}{10+10}$$

= 49.5+2.5

= 52

Mode can also be calculated using the relation. Mode = 3 Median-2 Mean. The mode is practically useful in cases where a measure most representative as a score has to be calculated.

Percentiles

In median the group is classified under two halves (the lower and upper) with regard to the phenomenon studied. But dividing the group into only two halves will not give a precise placement of the individuals. On the other hand, if the same group is divided into four equal quarters and the score at the end of each quarter is determined, the placement will be more precise. In the same way, the group can be divided into any number of equal subgroups and measures differentiating each group can be determined. When a group is large, it can be divided into ten or some times twenty subdivisions arranged in the order of magnitude of the scores. When the subdivision is ten in number, each subgroup will contain ten percent of the total group. Then the scores at the end of 10%, 20%, 30% ....up to 90% of the cases can be calculated. These scores are known as 'tenth percentile' 'twentieth percentile' .... 'ninetieth percentile' respectively. If the tenth percentile of the scores of an achievement test is found to be 20.5, it means 10% of the examines who took the test have scored marks below 20.5, while 90% of them have scored marks above it.

If a group is divided into four quarters, the score at the end of the first quarter is the twenty fifth percentile, that at the end of second quartile will be the

fiftieth percentile and that at the end of the third quartile is the seventy fifth percentile. They are known as 'first quartile', 'second quartile' and 'third quartile' respectively. The second quartile or the fiftieth percentile is the median. The tenth percentile is indicated as P_{10} P_{25} is equal to Q_1 and P_{75} is equal to Q_3. The Median is equal to Q_2 or P_{50}. Px represents the xth percentile.

The calculation of percentile is similar to that of the median. The formula is

$$P_x = l_x + \left(\frac{\frac{x.n}{100} - M_x}{F_x} \right) xc$$

Where P_x = x the percentile

l_x = lower limit of the class containing the xth percentile

N = total number cases

M_x = cumulative frequency upto the P_x class

F_x = frequency of the P_x class

c = class interval

An example is shown here

Calculate P_{10} from the following table?

Scores	Frequency	Cumulative frequency
90-99	6	150
80-89	8	144
70-79	10	136
60-69	15	126
50-59	25	111
40-49	30	86
30-39	20	56
20-29	18	36
10-19	10	18
0-9	8	8
	N = 150	

By examining the cumulative frequency, it can be seen that P_{10} will be in the class 10-19. This is because 10% of 150 being 15, it will require 15 cases to reach P_{10} and this happens at 10-19. The lower limit of the class is 9.5, cumulative frequency upto that class is 8, the frequency of the class is 10 and the class interval also is 10. So tenth percentile is

$$P_{10} = l_{10} + \frac{\left(\frac{10N}{100} - M_{10}\right)}{F_{10}} xc$$

$$= 9.5 + \frac{\left(\frac{10x15}{100} - 8\right)}{10} x10$$

$$= 9.5+7= 16.5$$

From this it can be concluded that out of 150 cases, one tenth, that is 15 cases have scored below 16.5 and the remaining 90%, that is 135 students have scored above 16.5.

Dispersion

The measures of central tendency help in getting an idea of the average performance of a group in some desired phenomenon. But this need not give an exact picture of the nature of the distribution of scores. Moreover, when two groups are compared merely on the basis of the average, there is possibility of being misled to incorrect judgments. So along with the measures of central tendency there is a need for examining the nature of dispersion of scores. In other words it has to be examined whether the scores cluster very close to the central scores or widely distributed around them. The statistical measures used for determining the nature and extent of dispersion are known as 'Measures of dispersion'. The different measures of dispersion are

(i) Range

This is the difference between the highest and lowest scores in a distribution, considering both these scores. Thus range is equal to H-L+I, where H is the highest and L is the lowest scores. 1 is added to give scope for including both the ends scores in a range. In some cases, range is equal to H-L only. Though it is easy to calculate range, it is not a good measure of dispersion, because it takes into consideration only the end scores.

(ii) Quartile deviation or Semi- inter quartile range

The quartile deviation is one half the scale distance between the 75th and 25th percentiles in a frequency distribution.

$$Q = \frac{Q_3 - Q_1}{2}$$ where,

$$Q1 = l_1 + \frac{\left(\frac{N}{4} - M_1\right)}{F_1} xc$$

$$Q3 = l_3 + \frac{\left(\frac{N}{4} - M_3\right)}{F_3} xc$$

l_1 and l_3 – exact lower limits of the interval in which the quartile falls.

m_1 and m_3 – cumulative frequencies up to the interval which contains the quartile.

c – width of the class interval.

In a normal distribution Q_1 and Q_3 will be on either side of the median at equal distance. Hence in such a distribution, the semi-inter quartile range will indicate the measure of the dispersion of the middle scores on either side of the median.

(iii) Mean Deviation or Average deviation

The average deviation is the mean of the deviation of all of the separate scores in a series taken from the mean. The deviation of a score from the mean can be either positive or negative, but since both indicate a deviation from the mean the sign is not considered.

If MD represents the mean deviation

$MD = \frac{\Sigma / fx /}{n}$ (where x represents deviation of a score from the mean)

In a frequency distribution

$$MD = \frac{\Sigma / fx /}{n}$$

Standard Deviation

The procedure involved in the calculation of standard deviation for ungrouped data is as follows.

(i) Calculate the mean

(ii) Take the deviation of the scores from the mean. Let this be x.

(iii) Square each deviation and find the sum. That is $\sum x^2$

(iv) Divide the sum by the total number of cases to get the average of the squares of the deviations $\frac{\Sigma x^2}{n}$

(v) Determine the square root of the above That is $\sqrt{\frac{\Sigma x^2}{n}}$

which is the standard deviation.

The symbol to represent standard deviation is σ.

Calculation of standard deviation from frequency distribution involves the following steps.

(i) Assume the midpoint of any convenient class as the mean.

(ii) Note down the class deviations on either direction of this class. Let it be 'd'.

(iii) Multiply each class deviations by its frequency and find their sum $\sum fd$.

(iv) Find the square of each deviation by multiplying each 'fd' again with 'd';

(v) Find the sum of all these squares of the deviations is $\sum fd^2$

(vi) Calculate the standard deviation using the formula $\sigma = c \times \sqrt{\frac{\sum fd^2}{N} - \left(\frac{\sum fd}{N}\right)^2}$

where c is the class interval.

An example is worked out for Quartile deviation, Mean deviation and Standard deviation.

Scores	f	mid x	cf	d	fd	fd²	$\lvert x \rvert$	$\lvert fx \rvert$
70-74	3	72	80	+5	+15	75	18.2	54.6
65-69	5	67	77	+4	+20	80	13.2	66
60-64	8	62	72	+3	+24	72	8.2	65.6
55-59	10	57	64	+2	+20	40	3.2	32
50-54	10	52	54	+1	+10	10	1.8	18
45-49	17	47	44	0	0	0	6.8	115.6
40-44	12	42	27	-1	-12	12	11.8	141.6
35-39	10	37	15	-2	-20	40	16.8	168
30-34	3	32	5	-3	-9	27	21.8	65.4
25-29	2	27	2	-4	-8	32	26.8	53.6
Total	80				40	388		780.4

Arithmetic Mean, x = 53.8

$$Q_1 = 39.5 + \frac{(20-15)}{12} \times 5 = 41.58$$

$$Q_3 = 54.5 + \frac{(60-54)}{10} \times 5 = 57.5$$

$$Q = \frac{(Q_3 - Q_1)}{2} = \frac{57.5-41.58}{2} = 7.96$$

$$MD = \frac{\sum |x|}{N} = \frac{780.4}{80} = 9.76$$

$$SD, \sigma = c \times \sqrt{\frac{\sum fd^2}{N} - \left(\frac{\sum fd}{N}\right)^2}$$

$$= 5 \times \sqrt{\frac{388}{80} - \left(\frac{40}{80}\right)^2}$$

$$= 10.7$$

Correlation

A teacher will have to handle a number of variables such as achievement, intelligence, aptitude, interest, socio-economic status and so on. Some of these when taken in pairs may be showing some significant relation while certain others may not be related in any way. The relationship between a pair of variables is denoted by the term "correlation". The extend of correlation will be different for different pairs of related variables. In some cases it will be so definite that it may be possible to represent the relation by a mathematical formula. When two variables are related in such a way that when one of the variable increases or decreases there will be an equal amount of change in the same direction in the other variable the two variables is said to have a "Perfect Correlation".

On the other hand when one of the variable increases the other tends to increase and as the first decreases the second variable tends to decrease but not as equal as the first variable, the variables are said to be positively correlated.

When one of the variables increases the other tends to decrease and when the first variable decreases the second tends to increases the variables are said to be negatively correlated.

If we consider a pair of variables where there is no relationship, the variables are said to be "Uncorrelated".

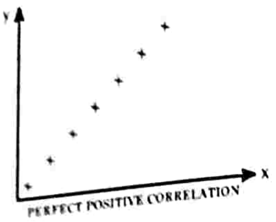
The extend or degree of correlation between a pair of variables is determined in the form of a ratio or fraction. The ratio indicating the degree of correlation is known by the term " Correlation coefficient". Perfect correlation is indicated by +1 or -1 for positive and negative correlation respectively. The complete absence of correlation is shown by a coefficient which equals zero. Thus correlation coefficient vary from -1 to +1.

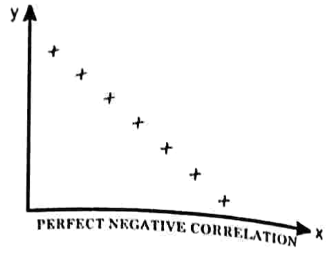
Graphical representation of Correlation

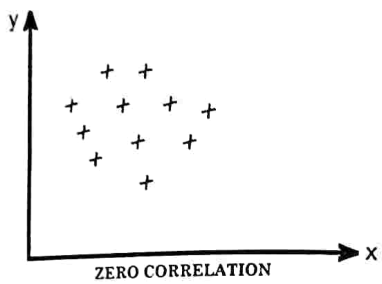
Different types of correlations can be shown graphically.

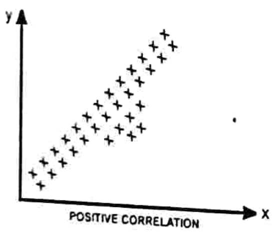
Coefficient of Correlation by Pearson's Product-Moment Method

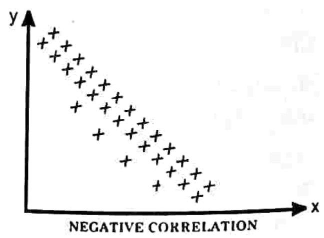
In this the product of the deviations of the X scores and Y scores from the means in taken as the basis for calculating the coefficient of correlation which is repre-











sented by the symbol 'r' and known as Pearson's 'r' in honour of the person who developed the method.

The steps involved in this are as follows.

(i) Determine the sum of the squares of x-scores $(\sum x^2)$

Where x = X-Mean

(ii) Determine the sum of the squares of y-scores $(\sum y^2)$

Where y = Y-Mean

(iii) Determine the product of each of the corresponding pair of x-score and y-score and find their sum $(\sum xy)$

(iv) Calculate the co-efficient of correlation using the formula

$$\gamma = \frac{\sum xy}{\sqrt{\sum x^2 x \sum y^2}}$$

A worked out example is shown below.

Subject	X	Y	x	y	x²	y²	xy
A	50	22	-12.5	-8.4	156.25	70.56	105.00
B	54	25	-8.5	-5.4	72.25	29.16	45.90
C	56	34	-6.5	3.6	42.25	12.96	-23.40
D	59	28	-3.5	-2.4	12.25	5.76	8.40
E	60	26	-2.5	-4.4	6.25	19.36	11.00
F	62	30	-0.5	-0.4	0.25	0.16	0.20
G	61	32	-1.5	1.6	2.25	2.56	-2.40
H	65	30	2.5	-0.4	6.25	0.16	-1.00
I	67	28	4.5	-2.4	20.25	5.76	-10.80
J	71	34	8.5	3.6	72.25	12.96	30.60
K	71	36	8.5	5.6	72.25	31.36	47.60
L	74	40	11.5	9.6	132.25	92.16	110.40
	750	365			595	282.92	321.5

Mx = 62.5 My = 30.4

$\sum x^2 = 595 \quad \sum y^2 = 282.92$

$\sum xy = 321.50$

$$r = \frac{\sum xy}{\sqrt{\sum x^2 x \sum y^2}} = \frac{321.50}{\sqrt{595x282.92}}$$

= 0.78

Correlation Coefficient by Rank Difference method

(Spearman - Brown Formula)

In product - moment method, the size of each score is the controlling factor in determining the magnitude of correlation. But there may be occasions when exact

scores are not available, or only the rank order is important. Hence, a technique for determining the coefficient of correlation from ranked data becomes necessary. On of the technique for determining coefficient of correlation by rank difference method is with the formula,

$$\rho = 1 - \frac{6\sum d^2}{N(N^2-1)}$$

Where ρ (rho) = correlation coefficient

d² = sum of the squares of the differences in ranks

N = number of pairs of scores

For example

Subject	X	Y	(R1) Rank of X	(R2) Rank of Y	(R_1~ER_2) = d	d²
A	35	40	9	7.5	1.5	2.25
B	39	35	7	10	3	9
C	54	48	5	5	0	0
D	65	65	3	2	1	1
E	45	45	6	6	0	0
F	60	60	2	3	1	1
G	40	40	9	7.5	1.5	2.25
H	75	75	1	1	0	0
I	38	38	9	9	0	0
J	57	57	4	4	0	0

$\sum d^2$ = 15.50

$$\rho = 1 - \frac{6x15.5}{10x99}$$

= 0.91

The above correlation is positive and high.■

CHAPTER-15

THE TEACHING OF ARITHMETIC

The word 'arithmetic' is derived from the word, which the Greeks used to contrast the science of number with the part of computing.

Historically arithmetic developed out of a need for a system of counting. The teaching of arithmetic has two major responsibilities: (1) the building up an appreciative in the fundamental processes which history has provided as a means for the efficient use of this system; (2) the socialization of number experiences that may contribute to the improvement of the common thinking practices of the race.

The arithmetic teacher must have a clear understanding of the place of arithmetic in the everyday attairs of adults, in specialized vocations that depend on technical expertness, and in the daily experiences of his pupils.

Arithmetic for Life

Arithmetic helps a man in his daily life affairs. No

exact answer is possible to the question: How much arithmetic does the average citizen use during the course of a normal day. Mr. A's quantitative description is height 1.63 meters, weight 68 kg., shoes sizes 7, sock size 11, body temperature 37 C, eyeglass numbers _2 and +1. His food is balanced in vitamins, mineral salts, proteins, carbohydrates, etc in specific proportion (in percentages or weights). He reads in newspapers that a 'road has been built to cut distance by 24 kms, a dam has been built to store 36,800 hectere-metres of waters, an express train has been started to some time by 2 hrs. 30 mts., price index has risen by 6 points, and look at a number of graphs showing the line of development. His purchases, salaries, payment of rates, taxes and bills, and the effect of variation of any of these on his budget are the arithmetical problems he has to face every moment. He wants to purchase a television set: what should be screen size? 45m or 60 cm? In his office he finds that production has dropped by 15%. What is its effect in quantity?

Arithmetic for vocations

Even common vocations like carpentry require applications of common measures, understanding number and computing ability. Housewife, retailers, bus driver, painter, gardener, etc., all use arithmetic. Some specialized vocations such as farming, business, electrician, mechanic, builder, nurse make special uses of arithmetic principles.

Arithmetic for continued learning

Algebra, geometry, trigonometry and physical and social sciences all call for understanding of arithmetic and computational ability. Arithmetic enters into study of subjects like geography, history, economics, art, music and biology. Specialized professions of engineers, doctors, chemists, physicists, astronomers, geologists, psychologists, banking, insurance, business administration all depend on the knowledge of arithmetic.

Cultural Value

On account of its very nature arithmetic has a real disciplinary value. It is definite— a sum must either be right or wrong; it obliges the child to concentrate and to reason. In short, it provides endless exercises in logic. It has a value in connection with man's life, with his physical environment, with the development of his occupations and with his advance in science.

Principles Underlying the Arithmetic Curriculum

1. The learning of arithmetic programme is a gradual growth process that should be guided at all stages by a systematic planned programme.
2. The arithmetic program should include a well-integrated treatment of the mathematical and social phases of the subject.
3. The content of the curriculum should be based on personal and social needs emerging in current living both in and out of school.
4. Arithmetic instruction should be done in close association with all school work.
5. A most fruitful approach to the enrichment of arithmetic instruction is the consideration of significant problems that will illuminate the present social situation for the learner, particularly in the area of economic competence.
6. Systematic provision should be made for adapting instruction to differences in the interests, abilities and needs of the pupils as well as differences in the rates at which they learn.

7. The curricuam should be so arranged as to provide for continuity of child development, with a minimum of strain and tension.

To increase the effectiveness of the plan, the following devices should be employed:

1. All arithmetical operations should be performed by all pupils individually, not by one pupil for the class or by teacher. Each pupil must assume the responsibility for the correct result in a reasonably brief amount of time.
2. Neatness and accuracy must be insisted on in all written work. Arithmetical computations that cannot be performed mentally should always be worked out on the page of the note-book on which the problems are written. Loose paper, or 'scratch paper', encourages carelessness. The pupil should always be made to feel that the written computation is just as important a part of his note-book work as the algebraic or geometric problems.
3. Certain processes need to be retaught more than once and certain difficulties need to be explained repeatedly.
4. Critical and frequent inspection of the pupil's written computations must be made by the teacher during the study period.
5. Occasionally tests must be given to determine whether the processes taught are retained by the pupils.

The First Four Fundamental Rules

The very first point to be remembered while teaching the four rules to the children is that the first four rules should be closely connected together and their operations taught side by side. The four fundamental process in arithmetic are merely four different ways of counting. Addition is counting forward, and subtraction counting backward. In multiplication or division we count forward or back-ward by leaps of uniform length.

In teaching addition and subtraction, it is better if the children are made to experiment with sticks, stones and pebbles, etc. It is also good if the children count on their fingers and toes. But such counting should not be stressed too long, as it practice in setting down numbers in the columns of units, tens and hundreds, etc. They can use bundles each containing ten sticks. Addition of small numbers can be performed with the aid of sticks, counters and ball frames.

The ideas of subtraction and addition arise at the same time, the one being the inverse of the other. The notion of 9 and 4 together making 13 involves the notion of 13 being 4 more than 9, and 9 more than 4. Indeed to add together 9 and 4 and to find the difference between 13 and 9 is the same process of counting from 9 up to 13, only in the first case the final attention is on the aggregate 13 and in the latter on the difference 4. Thus subtraction is the process of finding the number which added to the less will make the greater.

Addition

The Basic Facts of Addition

The basic facts in addition consist of all possible arrangements of two one-figure numbers from 1 to 9, inclusive. Since addition and subtraction are opposite processes, there is a related basic fact in subtraction for each basic fact in addition and vice versa. There are 81 basic facts in each process which do not involve zero. When the zero facts are included, there are 100 basic facts in each of the two processes. There are six stages in the sequence of experiences for teaching a basic fact.

1. Show the fact in concrete or in picture from.
2. Objectify the fact with markers or objects.
3. Reproduce the fact by drawing.
4. Write the fact in symbolic from.
5. Verify the fact by use of previously known facts.
6. Use the facts in problems.

Such things as toys, books, pupils, coins, and other classroom materials represent concrete objects for showing a basic fact. Dramatization is a very effective way to show a fact in concrete form.

A pupil has mastered a basic fact in addition when he has the following knowledge and abilities.

1. He can represent the fact with concrete objects.
2. He knows that addition means putting numbers together.
3. He can reproduce the fact promptly and confidently by dramatization, by a marker, or on an abacus.
4. He discovers that interchanging the position of the number does not change the sum.
5. He knows how to write the fact in both vertical and horizontal forms.
6. He can verify the result by use of other known facts.
7. He can use the fact in a problem.
8. He can give the sum quickly and confidently.

First practice should be given in small oral sums involving concrete quantities of every day use and then afterwards with abstract and large number.

1. Addition of numbers of one digit.

First: 3 pencils+4 pencils= 7 pencils.

3 inches+ 4 inches= 7 inches with the help of rulers.

then = 3+4=7

2. Addition with compound numbers involving digits less than ten. First take up those compound numbers which do not involve 'carrying' then those which involve 'carrying'.

Example 1:

Rs.	P.
6	2
4	3

2.

Tens	Units
5	2
4	3

3.

Tens	Units
5	6
3	7

We can add P to P and Rs. to Rs.

Similarly units to units and tens to tens, etc

Plenty of practice should be given.

Practice should first be given in the addition of number by counting forward and backward and backward by 1's, 2's, etc. and then latter arranged in vertical columns and in horizontal lines. The first step in adding two digit numbers should be as follows:

3+2= 13+2= 23+2=

Also 8+5=(8+2)+3=10+3=13 making up tens.

Similarly addition can be taught by actually adding rupees, paisa, etc. and then giving them the idea of adding units, tens and hundreds, etc.

Children should learn to check their work. If they add from the bottom upwards the check should be made by adding from the top downwards.

Subtraction

Just as addition means to put numbers together, so subtraction means to separate numbers or to take one number from another.

The idea of simple subtraction may also be approached through easy experiments with beads, ball-frame and sticks along the same lines as in addition. Subtraction should be taught with addition in the early stages. Number group cards and later simple cards with equations using both the simple subtraction and inverse addition are very useful for individual practice.

Take away	Fill in	Fill in
8 – 4	9 – ? = 3	? - ? = 3
8 - 3	8 - ? = 5	? - ? = 4

The essence in subtraction is to find the difference between two numbers. You may have to find the remainder or you have to make up a deficiency.

There are four different kinds of situations which give rise to subtraction, which may be shown by the following problems:

1. X has five marbles but he lost 3 of them. How many marbles are left with him?
2. X has 5 marbles and Y has 3. How many marbles has X more than Y has?
3. X has 3 marbles. How many more does he need to have 5 marbles?
4. X has 5 marbles, some of them black, the other white. If 3 of them are black, how many are white?

There are two methods for subtraction.

1. The Decomposition Method

Suppose we have to subtract 157 from 423

423=400+20+3=300+110+13

157=100+50+7=100+50+7

200+60 + 6 =266

We cannot subtract 7 from 3. Add one ten from the tens in the top line 13 and 7 from 13 leaves 6; we cannot take five tens from one ten, so we add 10 tens making 11 tens in the top line, 5 tens from 11 tens leaves 6 tenes 100 from 300 leaves 200. It is the easier method to understand and to begin with.

2. The Method of Equal Additions

423=400+20+3

157=100+50+7

200+60+6=266

We cannot subtract seven from 3, add ten units to the top and 10 to the bottom, seven from 13 leaves 6 and six tens from 2 tens we cannot subtract, so add 10 tens to the top line making 12 tens and add 100 to the bottom; six tens from12 tens leaves 6 tens; 200 from 400 leaves 200. This method leads to greater accuracy and greater speed. It cannot be made as meaningful to pupils in the primary grades.

Dr. Ballard is of the opinion that the method of equal addition resulted in greater speed and accuracy. For the teacher, of course, method of decomposition is easier to explain although it is confusing and difficult in cases like 100-28. It is used much more widely by adults than the method of equal additions.

The habit must always be formed of checking subtraction sum by the corresponding of equal additions.

Multiplication

Multiplication is defined as the shortened addition of equal quantities. The following order for learning the multiplication tables would be helpful in many cases; the tables of 2's, 5's, 10's should be taught first, then 4's,8's, 3's, 6's 9's and last of all 7's. Whichever order is taken the tables should be connected with one another as much as possible.

1. The tables should be built up by the children themselves with the aid of a variety of apparatus such as sticks, beads, ball-frame and so on.
2. While building up tables, pupils should use them in multiplication and division practical examples. The tables must become automatic by drill and repetition. Repetition is necessary but it is much more effective when it is thoughtful and purposeful.
3. The pupils must be made to realize that the product of any two numbers is always the same, whichever is made the multiplier, e.g., 2 x 9 = 18 and 9 x 2 = 18 [Commutative law].

Multiplication is addition in groups of more than one. The order of factors does not affect the product. It is a quick way of adding several equal numbers since multiplication and division are short forms of addition and subtraction respectively. There is the same number of basic facts in each process, a total of 81 facts. The teaching of multiplication is greatly simplified if it is divided as follows:

1. The multiplicand consisting of units and tens, the multiplier of units only, without carrying over, Example: 14x2,24x2,21x2,etc.

2. The multiplicand consists of units and tens, the multiplier of units only, with carrying over.

   Example: 24x3; 36x2; 46x2

   632x 5; 848x6; 2348x7

3. In the multiplication of any number by 10 and 100, the puplis may realize by actually multiplying that by placing a zero to the right of any number increases its value ten times.

4. In multiplying by 20, 30, 40, etc. the pupil will be able to do so by multiplying first by ten and then by,2,3,4, etc.

5. Multiplying by numbers less than 100 say 56, we must multiply by 50 and 6 and add the products. Multiplying by 50 is just multiplying by 5and then adding a zero. To this add the product of 6. Thus in this method, we begin multiplying from the left. Moreover, we write the full products and do not omit the zeros.

The multiplication facts may be written in either the vertical from, as 5x2 =10 or in the equation from, 2x4=8. The pupil should read a multiplication fact in such a way which will convey the most meaning to him. The fact 2x5=10, should be read "Two 5's are 10', and not "two times five's (is or are)10"

The multiplication fact should be developed as a system but practiced in a random order.

The teacher should remember that zero may be multiplied by a number but reverse is not true.

A pupil understand and masters the combination. 3x5=15 when he knows how to

(a) Find the answer by adding three 5's

(b) Find the answer by adding five 3's

(c) Find the answer by relating it to some other fact involving 3 or 5,as 2x5.

(d) Write that fact in either the horizontal or the vertical form.

(e) Use it in problem.

(f) Relate it to division.

(g) Give the answer promptly and confidently.

Division

Division is subtraction in groups of more than one it is a quick way of subtracting equal numbers. It is the reserve process to multiplication. There are two aspects of division.(a) Partition or sharing. i.e. to determine the size of each part, when a given quantity is divided into a number of parts. (b) Quotition or measuring- finding out the number of times of quantity contains another, e.g; how many boxes each containing 20 marbles can I fill from a pile of 180 marbles.

While teaching the four fundamental operations keep in view:

(1) That these may not be taught by rote method, this simply encouraged passivity and its consequent evils rather than make the pupils discover and formulate the rules and assimilate them by repeated exercises and verification.

(2) Problems should arise wholly out of concrete situations and should be in concrete way. No formal teaching with the help of rules may be depended upon.

(3) It is an erroneous conception to say that it is better to deal with addition thoroughly in the first class and then proceed to subtraction and finish it completely. It is useful that all the opera-

tions may be dealt with in the first class because when we give 6 toys to a child and ask him to share equally with his brother, he feels no difficulty in doing this, he makes use of three process individually, subtraction, multiplication and division.

6-3=3; 2x3=6; 6 ÷2 = 3.

(4) Whenever a new principle is learnt, it should be used in as many different ways and situations as it is possible for it to be used.

As soon as a child has realized the connection between multiplication tables and simple division sums he should be given a card of the type shown for oral work.

Multiplication	Division
8x3=	24÷3=
7x4=	28÷4=
9x2=	18÷2=
6x7=	42÷6=

Partition: Sharing, Dividing into, between or among:

Divide 28 in to four equal parts. How many in each part? What is one quarter of 28? Five yards of cloth cost Rs.35. How much does 1 yard cost?

If 12 apples are shared among 3 boys, how many will each get?

In all examples of partition we are given the number of groups and are required to find their size .i.e., of how many or how much each group consists. Partition is really fractional division, for when we say 'Divide 12 among 3 boys' we mean 'Give a third of 12 to each boy'.

(2) **Quotition:** Measuring, Grouping, and Dividing. How many times can I subtract 5 from 35?

How many fives must I take to make 35?

When 35 is divided by 5. What part of 35 is 5?

How many rupees in 268 Paisa?

I have 12 apples. To how many boys can I give three? We may regard question as measuring up one quantity by another quantity, or as continued subtraction.

Given that 9x4=36, and using only these three numbers write down as many equations as you can. Results:

¼ of 36=9

1/9 of 36=4

36÷9=4

36÷4=9

4x9=36

Decimal Fractions

It is a known fact that pupils and teachers are ordinary shy of decimals and in view of the introduction of decimal system in money, weights, etc, it is essential to make the working by decimals easy and cued. We are used to the convention of simple decimals into complicated vulgar fractions. If the notion of place value of digits is correctly fixed and if the decimal point is taken as an index of limit between integral and fractional part of a number, the computations of decimals would be easy.

The operations with decimal fractions involve little difficulty as compared with vulgar fractions. Engineers and surveyors have long recognized this advantages and are using rules and steel tapes that are graduated in tenths and hundredths of a foot.

Studies relating to the teaching of decimal fractions have shown that the greater source of error of pupils of the secondary school is the wrong placing of the decimal point in computations.

Decimals were invented in order to make fractions unnecessary so that the work could be done easily.

When the pupils of the primary classes are learning how to measure lengths less than an inch the idea of decimals should be introduced. An inch is generally measured by the number of tenths in it, we can find out a method of expressing 5/10, inch or 3/10 inch in a certain notation. The decimal concept may also introduced with our money system.

For a first lesson in decimal I would suggest taking metre rod divided into tenths.

A decimal fraction is a fraction whose denominator is never written and the unwritten denominator is limited to some power of 10. Just as integer digits have a definite value by virtue of their position so the numerator digits of fractions are by their positions given value as tenths, hundredths thousandths, etc.

Represent 1, 111 on the blackboard thus

Thousands	hundreds	tens	units
1	1	1	1

The value of each digit increases ten times as it moves each place to the left. In the reverse order the value decreases ten times as each digit moves of the right.

If we continue moving the digit to the right beyond the units, the value of the next 1 would not be a whole number but 1/10 of a whole; similarly the next one would be ten times less, i.e. 1/100th of a whole. We can thus write:

Thousand	hundred	tens	unit	tenth	hundredth etc.
1	1	1	1	1	1

These may be expressed in fractions as follows

1000	100	10	1	1/10	1/100 etc.

If the headings and place value were omitted, we would not be able to know the units figure. The position of the unit can be indicated by a dot called the decimal point which is placed directly after it to the right. Hence 64/10inches may be read as 6 point 4 inches written as 6.4 inches.

The Metric System

The extension of the decimal system to money and measurement, known as the Metric System, was introduced in France at the time of the French Revolution. In 1790 the French Government appointed a commission of scientist to frame a new system of measures and to determine a unit of length. They decided that the latter should be 1/10,000,000 of a quadrant of the meridian. This length, based on a measurement between Dunkerque and Barcelona, was called a meter; its British equivalent is 39.3708 in. It has since been found that the measurement of the meridian was not quite accurate, so the meter is now defined as the length of a standard bar kept in Paris.

It is maintained by some educationist and it is almost a truism to say that whether the point of view be that of science, of business or of teaching; the metric system of money, weights and measures is unmistakably superior to that previously prevalent in our country.

The mathematics of the metric system being precisely that of decimal fractions. It can be taught with ease and requires no additional time. The ease and efficiency for teaching of decimals is considerably increased because of use of the decimal weights and measures in actual life.

Why Decimal Coinage

The Government of India introduced decimal coinage in India with effect from April 1957. According to this system the rupee which remains the same in value and nomenclature has been divided into 100 equal parts called Paisa instead of its previous division into 192pies. The other coins in the series are 2,5,10,25 and 50 Paisa instead of 2 pice 1 anna 4-anna and 8-anna pieces.

Decimalization of coinage has been acclaimed all over the world as the simplest form of coinage, making calculations easy and quick. In our modern complicated system of trade and commerce, easy conversion of money is most essential and decimal system which works on multiples of ten makes the task of conversion easier.

The volume of opinion in favour of decimal currency has since increased and in 1955 the government of India introduced a bill in the Parliament on the subject. This bill became an Act in September 1955 authorizing the Government of India to introduce decimal coinage in the country.

Position in other Countries

With the introduction of decimal coinage, India has carried out a reform in currency which has already been adopted in 105 out of about 140 coin-issuing countries of the world.

Decimal coinage, in order to be fully effective must be linked up with the introduction of the metric system of weights and measures. The Government of India have already taken a decision regarding the latter reform which will be spread over a period of 15 years. Thus, decimal coinage which had already been introduced in 1957 was the precursor of the bigger reform of standardizing weights and measures in which there existed an enormous multiplicity and variety all over the country leading to a great deal of confusion. Metric system of weights and measures has been introduced with effect from April 1, 1960 in the country.

Although the new coinage will be on decimal basis, it will be free from any orthodox rigidity involved in a meticulous application of the decimal principle. According to the orthodox definitions, decimal coinage is a currency in which the various denomination of coins are arranged in multiples or sub-multiples of ten (Latin 'decem') with reference to a standard unit be the higher coins will be 10, 100, 1000, etc. and the lower units will be.[1] In a perfect system there would be no breaks or Interpolations, but the actual decimal currencies adopted in various countries do not show this rigid symmetry.

In India, we are having the 'Paisa' as the unit, with its multiples of 10 and 100 but there are also other multiplies of 2,5,25 and 50. In this way, we are retaining some of our familiar coins, ensuring conversions at the intermediate stages and at the same time deriving the advantages of the decimal system.

Ratio

Take 12 beads of the same colour and perform the following experiments:

1. Place one bead on one side of the table and, the remaining 11 beads of the other and ask how many times this one bead is present in the group of 11 beads.
2. Place two beads on one side and the remaining 10 on the other side ask how many times the two beads are present in the group of ten.
3. Place 3 beads on one side and the remaining 9 on the other side and again ask how many times 3 beads are present in the other group.
4. Again place 4 beads on one side and remaining 8 on the other side and ask how many times the cluster of four is contained in the other cluster.

In the above cases we have compared two quantities as to how many times one quantity is contained in another. This relation is called the ratio. Thus the ratios, in the above are respectively 11: 1;5: 1;3: 1,2: 1.

After this the ratio may be found between 5p. and 15p; 2p; and 20p. and so on.

The symbols for ratio are –, and: as 3/5 and 3:5. The ratio of one number to another is the quotient of the first number divided by the second. The two numbers used in a ratio are called the terms of the ratio. The first one is named as the antecedent and is the divident, the second is named the consequent and is the divisor.

Two quantities can be compared if they are of the same kind or if they can be expressed in terms of the same unit.

A ratio is the comparison of two like quantities by division.

Questions of the following types may then put to the pupils:

(a) Divide Rs.20 between two boys in such a way that if the younger one gets Re.1, the elder one may get Rs.4.

(b) Divide Rs, 100 in the ratio of 2:3 between A and B. Verification of the answers must be done side by side.

Proportion

When four quantities are so connected that the ratio of the first to the second is equal to the ratio of the third to the fourth, they are said to be in proportion. The order of sequence of terms of ratios is very important.

For instance, 5:15=4:12. A statement in proportion is a statement of the equity of two ratios. It may be written a 5:15::4:12 or 5/15=4/12.

The first and the fourth terms are called the extremes and the second and third terms the means or middle terms. 12 is called the fourth proportional.

The product of the extremes is equal to the product of the means.

Then 4x10=5x8

Three quantities of the same kind are said to in continued proportion when the ratio of the first to the second equals the ratio of the second to the third The second quantity is called the mean proportional and the third quantity is called the third proportional.

8,4 are in continued proportion if 16::8: : 8: 4;8 is the mean proportional between 16 and 4 and 4 is the third proportional to 16 and 8.

The mean proportional between two numbers is equal to the square root of their product.

The notion of two quantities increasing and decreasing together is used in cases such as the following:

1. Costs and numbers
2. Mileage and fares
3. Time and distance
4. Circumference and radius, and
5. Time and work, etc.

The close connection of the numerical idea with the geometrical representation is visible in the fact that all the above can be easily represented by means of graphs and all of them give us a straight line graph.

Proportion may be simple or compound.

Simple. If 6 books cost Rs 14, find the cost of 9 books.

Compound. If 9 mean can reap 70 acres in 16 days, how many acres can 24 men reap in 30 days?

Inverse proportion

The students may be made to distinguish clearly between direct and inverse proportion. Concrete examples may be given so that they grasp the idea clearly. For instance, in the case of Area of a rectangle, A=l x b where A=Area, l=length, b= breadth; A varies according to b or l changes. This is the case of direct proportion. In the later proportion we have pressure= K/volume (Boyle's law)

Time period $= 2\lambda\sqrt{length / gravity}$

If one increases, the other decreases.

3 lbs, of tea cost Rs.8. How many lbs. will cost Rs.24?

Lbs	Rs
3	8
X	24

6 men can do a piece of work in 20 days, how many could do it in 15 days.

Men	Days
6	20
X	15

There are some problems which are quite absurd and vague and hence are unacceptable.

1. It takes 3 minutes to boil 5 eggs. How long would it take to boil 15 eggs?
2. My son weighed 20 lbs when he was 2 years old. How much will he weigh when he is 40 years old?
3. A rope stretches ½ cm when loaded with 1cwt. How much will it stretch when loaded with 20 cwts?

Percentage

Without some understanding of percentage it would be impossible to calculate interest, determine discounts and make simple routine comparisons of quantitative data. Even the intelligent perusal of the daily paper requires at least an elementary understanding of percentage. This has many direct applications to the ordinary affairs of all people, such as all advertisements dealing with percentage, sales clearance, marks in the examination, charts showing the relationship of fractions and decimals, trade bills showing discounts and charts showing increase in price and decrease in quantity and vice versa.

This topic should be introduced to the class through its use in daily life.

Suppose you want to purchases a number of books say, worth about Rs.200 from a certain bookshop. The shopkeeper in order to attract you will take something less. He says that he will charge Rs.90 for every Rs. 100 ie, he leaves Rs. 10 per Rs. 100.

Again, suppose you want to sell your house for Rs, 5000. You try hard for a customer but no body pays you more than Rs. 3000. Then a person comes to you and says that he will find a customer for Rs. 4000 provided you pay him something. This money is promised to be given. Say it is Rs. 2 for every hundred.

Leaving of Rs. 10 and giving of Rs. 2 in the above two examples are examples of percent.

Percentage means per hundred. The whole of anything is 100 percent. Percentages are the same as fraction and decimals with 100 as unit of percent is merely a ratio or fraction expressed with a denominator of 100.

The main thing is to lay stress on the fact 'x% of a thing means x/100 of it. Percent is a very concise and simple way of expressing comparisons as, which is better 27 out of 60 or 19 out of 50. Secondly comparing fractions 5/6, 8/9 and 33/35, which is the biggest?

The necessity for reducing fractional amount of a quantity to the same denominator in order to compare their values should lead children to appreciate the advantages of choosing 100 as the standard denominator and so expressing a fraction as a percent. Comparison can be made in terms of fractions, decimals and percents. Decimal in hundredth is a percent.

The term percent is used in interest, profit and loss, discount, stock and many other topics: but whatever the use, the following are the situations which arise: (a) What percent of 50 is 2? (b) What is 2% of 50? (c) of what number is 2.50%? (d) What will produce 2 if it is increased by 50%? (e) What will produce 2, if it decreased by 50%? (f) Express 2/5 as a percentage. (g) what percentage of Rs.4 is Rs.2?

All problems in percentage can be reduced to one of the above types. And hence the teacher should enable the pupils to handle quickly the above types.

To make the pupils realize the value of percent, the teacher should take quantities, and data gathered from other subjects such as geography, science, hygiene, domestic economy, etc.

The pupil should become thoroughly familiar with the equivalent of the percents that are most commonly used.

The following illustrations are commonly used.

½ =0.5=50%

¼ =0.25=25%

¾ =0.75=75%

1/3 =0.333=33%

1/5 =0.2=20%

1/8 =0.125=12%

1/10 =0.1=10%

Percentage helps in the following:

1. Changing a common fraction to a decimal fraction.
2. Changing a decimal fraction to percent.
3. Changing percent to a decimal fraction.
4. Changing a decimal fraction to a common fraction.

Profit and loss

The students are familiar with the every day buying and selling of articles. They have also an idea of percent.

If I buy a pencil for 20p. and sell it for 25p. find my gain.? In this sum, the student will be able to find that the gain is 5p. The cost price is 20p. and the sale price is 25p. Similarly if a man buys a cow for Rs. 100 and sells it for Rs. 80 he loses Rs.20. So the loss is Rs. 20.It will not be difficult to make the pupils realize that;

Gain=selling price-cost price.

And Loss=cost price-selling price.

One thing that has to be made clear is that the profit or loss is always reckoned on the cost price and it is generally calculated in the form of percent which can be calculated easily by the unitary method. The idea of profit and loss in business is easily illustrated by business enterprise in which pupils often engage. Pupils should search for real up-to-date business practices and problems.

Average

Average is best introduced if an approach is made through the average performance of players in cricket, or through average marks earned in an examination or through average attendance, height, weight etc. Children can then be taken to average rainfall in the country, average temperature, average expenses on a picnic etc. They should also be made to measure and note the length of their average pace, average time taken to go to school etc.

A man earns Rs. 3 on the first day, Rs. 2 on the second day, Rs. 5 on the third day and Rs.6 on the fourth day. His total income of 4 days is Rs.16. It means the same thing as if he had earned Rs. 4 per day. This amount of Rs.4 is called the average value. Thus the average value of any number of quantities of the same kind is their sum divided by their number. It is called the mean value.

The sum of a number of quantities=

Their average x their number

To find the average 12", 15", 23", 9", 20" and 35" draw these lengths and bring them in one line. It will be equal to bringing 19" (their average) six times in one line. Average of to two or more numbers is some where between the smallest and the largest. Average is merely an indication of generality. If numbers are arranged in order, the average is somewhere about the middle number.

Interest- Simple and Compound

Simple interest, the students already know what 5% of a thing means. So long as one understands what percent means, he should have no difficulty in solving sums on simple interest. Of course, the idea of simple interest has to be introduced in the very beginning as to how one man may need money for some business while another man may have surplus money to invest, and so the latter may give his money to the former, and receive something extra called interest. Many people have savings accounts, and receive interest from the bank much information about interest can be collected by the children themselves. They may bring to school illustrations of loans, mortgages, savings accounts, bonds other interest yielding investments.

The students have had the idea of unitary method beforehand. As such they will be able to solve the following.

Find the simple interest on Rs. 200 for 2 years at 5% per annum.

Interest on Rs. 100 for one year= Rs.5

"Rs. 1 " = $\frac{5}{100}$

"Rs. 200" = $\frac{200 \times 5}{100}$

"Rs. 200 for 2 years = $\frac{200 \times 5 \times 2}{100}$

Similar examples may be solved on the blackboard, till the pupils realize that for finding simple interest of any sum for a given time at given rate we multiply sum with rate and divide by 100. So in generalized form, we can put it as follows:

$$S.I = \frac{\text{Principal} \times \text{rate} \times \text{time}}{100}$$

$$\text{Or } S.I = \frac{P \times r \times t}{100}$$

After this sufficient practices may be given in the application of this generalized rule. Later on inverse cases of finding principal, rate and time may be taken up.

Compound Interest

An idea of compound interest may be developed by telling the pupils that money is said to be lent at compound interest when at the end of a year, the due interest is not paid to the lender, but added to the sum lent and the amount thus obtained become principal for the next year. The same process i,e. addition of the yearly interest of the previous amount, is repeated until the amount for the last period has obtained. The difference between the original principal and the amount is the required compound interest.

Discount

This topic is closely related to the life of the home because many articles needed in the home are sold at discount during the year. The first problem is to make

clear what discount means and to show the variety of reasons for giving. Discounts which may be collected by the pupils and brought to school where they may be discussed. The topic should be discussed at several place of the course finally leading trade discounts.

People are buying and selling goods in the market on credit. Suppose I have to pay you Rs. 104 in a year's time for goods received on credit. But you want your money immediately. Certainly, I cannot give you Rs.104 in full at present. Let us go to some good bank, say, The National Bank Ltd, and enquire the rate of interest allowed by it. Suppose the rate is 4%. Clearly, I can discharge my debt but paying Rs. 100 at present. You lose nothing because if you deposit Rs. 100 in the bank, it will amount to Rs, 104 in a year's time. The amount deducted, is called the discount; Rs. 104 is called the amount the Rs. 100 is the present worth; (P.W)

Discount = Interest on P.W

Amount = P.W + Discount.

Area and Volume

Area. It is quite wrong to start with a definition. The pupils know that when they want to find length they have to see how many times it contains the length of one foot. In other words we choose a definite length as our unit and we see how many times that unit is contained in the length to be measured.

Now in order to measure the area of a rectangular black board, we must first choose some definite areas as our unit and then find how many times that unit is contained in the area of the black board. The teacher might suggest an idea of fixing a duster as the unit of area and thereby measure the area of the black board. But in view of the necessity for a fixed standard of area, we can't adopt a duster or a note -book or a book as a standard unit. A square foot, therefore is a better unit of area. Cut out a square foot of card board and see how many times it is fitted on the black board. Again draw a rectangle an the black board, say 5 m , by 3 m.

Mark the rectangle into square meters by the aid of a standard cardboard square meter and count how many square meters there are in the rectangle. In each row we have 5 sq. m. and there are three such rows . Therefore the whole area is 3 time 5 sq. m or 15 sq. meters.

After doing a number of similar examples, the class will be led to the following rule: "To find the area of a rectangle, multiply the number of units in the length by the number of similar units in the breadth, the resulting number is the number of square units in area."

Volume

The first thing is to develop the idea of volume or capacity just as we developed the idea of area. We have to go through the same steps: e.g., (i) choose a unit of volume, (ii) see how many times it is contained in the required volume. Suppose we want to find how much water a jug will contain and suppose we take a tumbler full of water as our unit of volume and thereby see how many times the tumbler full water is contained in the jug. So again there is the necessary for a fixed standard unit. A cubic cm or a cubic foot may be taken as the unit of volume. Now take a rectangular block., say, 5 cm by 4 cm by 3cm. Divided its base into square cm and mark it into layers each of 1cm. thick. In the bottom layer on each square cms in the area of the base and the same is true of each layer.

Hence the rule: To find the volume of a solid uniform figure, multiply the units of length, to those of breadth, and then to those of height of the similar units, resulting number will be the volume in cubic unit of the figure.■

CHAPTER-16

THE TEACHING OF ALGEBRA

Algebra has acquired a reputation, among teachers, pupils, and parents alike, as one of the most difficult and troublesome courses in the secondary curriculum. It presents a radically new and different approach to the study of quantitative relationships characterized by a new symbolism, new concepts, a new language, a much higher degree of generalization and abstraction that has been encountered previously, in arithmetic. Algebra is more concerned with the conscious examination and study of process than with particular answer to particular problems; and by the essential dissociation of many of its parts from institution and concrete experience.

Algebra, like arithmetic is a science of numbers with this distinction that the numbers in algebra are generally denote by letters instead of figures. There is a difficulty of finding a precise boundary between arithmetic and algebra due to the fact that the distinction between them consists not so much in difference of subject matter as in difference of attitude towards the same subject matter.

Algebra is often referred to as generalized arithmetic.

Hence whenever concrete quantities come under the domain of algebra, it is only numerical.

Algebra may be related to geometry by saying that algebra is only written geometry and geometry merely pictured algebra.

Aims and Functions of Algebra

(1) The aim of algebra is to correct the weakness and supplement the deficiencies of language as an instrument of abstract investigation and exact statement. Words and phrases, as the vehicles of ideas are replaced by symbols with a consequent gain in clearness and conciseness. A formula, consisting of an arrangement of symbols, is free from the ambiguity which often besets verbal sentence. Besides, algebra is more effective vehicle of complicated meaning.

(2) One of the most important aims of teaching algebra is to enable the students to use it for solving some difficult problems of arithmetic. Thus it aims at simplifying the work of the students by giving them compact formula or generalization to be used for solving difficult problems and to save their time. The solution of problems to equations and reduction of labour by judicious factorization, are examples of this. Algebra becomes a machine for facilitating both bare calculations and the solutions of complicated problems in all reign of thought in which number of measurement plays a part.

(3) Algebra provides a new and refined approach to the study of abstract mathematical relationship through the use of a new language and a new

symbolism. Algebra affords a compact symbolism in which we record what we decide, and by this very fact it compels or at least conduces towards an accurate analysis. Thus it aims to inculcate the power of analysis in those who study it and this power is the first and the foremost thing required for a student of mathematics.

(4) An algebraic statement one made, often poves to be capable of transformation into other forms which greatly simplify the calculation required. Every algebraical identity is shorthand for a sentence and this sentence contain an infinity of particular statements. It has been described as the most important labour saving device invented by man.

(5) Some regard algebra to be an instrument for mental training but it give something more than mental training. It has something definite to contribute to culture education otherwise chess also provides something towards mental training. Algebra is a great help to other sciences which are the foundations of all the progress that we find around us. It helps in the generalization of scientific truths into simple concise formula.

(6) Algebra is studied for its practical value. It is used in many of the trades industries.

(7) Checking the results is more simple and satisfactory process in algebra than in any other branch of mathematic. Thus it provide certainty of results and thus provide confidence among the students.

(8) Emphasize the inductive method. Introduce as far as possible every general method by concrete, ie., numeral examples. Thus before considering $a^m \times a^n$, find $2^3 \times 2^5$.

Algebra as compared with Geometry

Algebra requires the same accuracy of thinking, and the same, or possibly greater, accuracy of detail than geometry. It may be graded as perfectly, and its introductory chapter may be made even simpler than those of geometry.

On the other hand, algebra does not require as much reasoning and this reasoning is not always of the same high order as geometry. It requires a certain amount mechanical drill.

When should algebra be studied?

Any new topic of algebra or geometry should be studied whenever the necessity for it arises, and not before. Thus square root should be taught in connection with the Pythagorean theorem; similar triangles may be connected with proposition. All algebraic facts should as far as possible be illustrated geometrically, and vice versa.

Advantages: (i) It may arouse more interest than the customary mode. (ii) It may at some stages of the work show the student the necessity of studying certain topics. (iii) It may train the student better to apply his knowledge during the time when geometry alone is studied.

Disadvantages: (i) The complete merging of algebra and geometry may not be at all possible. (ii) Such course of study are still in a experimental stage. (iii) The text books are not written along these lines.

Beginning Algebra

The introductory period in beginning algebra is a crucial period for developing not only basic skill and knowledge but also important interests and attitudes. The methods and processes that the pupil is to acquire in algebra must, in this transitional period, be integrated with and built on his previous experience in arithmetic. While the idea of general number is being acquired, a beginning must be made in vocabulary building, in learning the symbolism, and in using the processes. At the same time the pupil must develop the appreciation of the importance of algebra to himself and to society.

Formerly the method of beginning algebra was to give a long list of definitions to the students to memorize them, but now this method is fast disappearing. The two modern methods of approach to the subject are(i) via the problem and the equation, and (ii) via the formula. Today the importance of the child is taken into consideration. His interest is to be aroused in the lesson first so that he may form a love for the subject.

It is not easy to decide whether problem or formula is likely to make this appeal more persuasive and teachers are divided on the subject.

The early introduction of problems seems, however to possess more advantages. It is in accordance with the historical development of the subject and it awakens a boy's interest by appealing to his constructive instinct. Also the boys will not find it difficult when they will meet with other problems at a latter stage and in the hands of most teachers we consider it as the easiest method. Further it is essential to bring about all the manipulation that is essential and that manipulation should grow gradually.

Graphs

Descartes is usually credited with the invention of graphs. A graph is a geometric picture showing arithmetical and algebraic relation.

The course of an illness, the constitution of the population, the efficiency of a petrol engine, all these and many more can be brought to life by means of a suitable diagram. The graphing of statistics should come first.

Thus graphs showing the exports and imports, family consumptions, rainfall of a place in different months, results to a school, etc..., etc., are some of the examples of simple graphs to be shown to the students in the early stages of introducing the topic.

Later on the arithmetical work should eventually lead to the construction of graphs in which the table of values obtained by calculations or from some simple experiment like measuring the length of a rubber cord from which various weights have been suspended. The law implied in this experiment can now be explicitly stated as an algebraic law of the type $L = a + bw$ where L = length of the cord when some weight is suspended, and W = weight and a, b are constants.

Coming to the drawing of graphs it may be added that after sufficient practices in arithmetical work we may turn to algebraic expressions. First of all we should take the equations like:-

$x = 5$; $y = 3x$; $5x - 4y - 7 = 0$ and later on like, $x^2+y^2 = 36$, $(x - 3)^2 + (y - 2)^2 = 25$, $y = 4x^2$ and so on.

Following are some of the points showing their importance.

(1) The graph makes an immediate appeal to the eye. The visual image is one, above all others, which can most easily be remembered, analysed and interpreted.

(2) The graph gives a picture of the variation of one quantity with another. It should give the idea of

the interdependence of two related quantities; it should give idea of continuity. It often enables the investigator to discover whether there is any variation or correlation or not.

(3) It enables the students to solve some difficult problems which otherwise they cannot; as solution of higher equations and transcendental equations etc.

(4) The students acquire a clear notion about functionality a very important concept.

(5) Students feel interested in it as it delivers its message in a form readily taken in by the eye.

(6) A graph gives excellent opportunities for insisting upon the 'great school virtues' of neatness, carefulness and accuracy.

(7) The graph also gives the pupil an opportunity of criticizing his own work.; a graph that suddenly moves off in an unexpected direction or a point that spoils the smoothness of a curve soon leads the pupil to suspect a mistake.

(8) The graphic representation of fact appeals to the aesthetic sense.

(9) A graph is pictorial representation of some relation given in the form of a verbal problem or an equation or a formula. It is a device of saving many laborious computations.

Understanding Algebra through Geometry and Arithmetic

Geometry and arithmetic can be used to illustrate and aid the understanding of many concepts in algebra. The difference between x^2 and $2x$ can be illustrated by a diagram for x^2 as a square with side the x units long and a line x + x or 2x units long. Through such diagrams the pupil learns to differentiate between a 'coefficient' and a 'power' of a quantity.

Many complicated problems in algebra can be made concrete by considering parallel cases in arithmetic.■

CHAPTER-17

THE TEACHING OF GEOMETRY

The word 'Geometry' originally meant measurement of earth. There are two types of geometry. The first is informal and the second is formal or demonstrative. Geometry has a double value, first as knowledge and second as a method of logical thinking.

Geometry is recognized as a study important for cultural development. It is the key to mathematical thinking. Its importance arise partly from its value in demonstrating the nature and power of pure reason. On the basis of a few axioms or assumptions, the student is able to erect a logical structure of established truths that can be used to discover and prove new facts. No other experience can demonstrate so clearly the meaning of mathematics as the science of necessary conclusions or reveal so effectively the power of human reasons. Geometry provides ideal field for observing and exercising the process of deductive logic. It provides a content that ranges from the simple to the complex; yet it is objective and non-controversial. The results are verifiable as correct or incorrect.

Technical advancements have placed an increasing importance on the geometry of form, size and position. Not only in engineering. machine shop, and construction industries, but in landscape, architecture, interior decoration and areas of appreciation.

General Teaching Procedures on Geometry

In all critical areas of teaching geometry, the problem is merely that of following the effective learning sequence. A setting is established in a concrete, significant situation: experience are provide to develop the generalization, and the concept is applied to new life situation.

The teacher must depend on first hand experience and visual aids. The blackboard must be used effectively for all theorems and originals. The figures must be sufficiently clear and accurate to avoid distortion. Colored chalks may be used to emphasis significant details. Blackboard instruments-compasses, protractor, and ruler-require skillful handing. The room should reflect the spirit of mathematics. Pictures, models, exhibits and the bulletin board should be brought into a unified setting. The setting should be varied from time to time. The class should come to feel a part of the responsibility for the setting. Slides, film strips and moving pictures serve a similar purpose, for they make up in economy of time what may be lost from first hand experience.

How to begin Geometry

Experimental stage

Early instruction in geometry should be to examine models and ask about their faces, the idea of direction etc.

(i) Object teaching through direct method by keeping an accurate record of pupil's work of the definitions which he has developed from the examination of various objects.

(ii) The pupil should be taught to express himself by drawing, by construction and by words as fully as possible.

(iii) The pupil should satisfy himself about the geometrical truths by drawing construction and superposition and not by logical proofs. Such an approach, through constructions, has the advantages of developing skill with instruments, an ability needed for constructions accompanying demonstrations throughout the course. It provide a review of definitions and concepts, and opens up the study of other units through a fresh and active approach to the subject.

(iv) The subject-matter should be developed through the combined effort of the pupil and the teacher through skillful questioning without the use of books containing definitions and demonstrations, in its relation to life.

(v) Accuracy and neatness should be insisted upon.

(vi) It should be emphasized through correlation, Arithmetic, algebra and geometry are not different from each other.

Arithmetic is the science of numbers, algebra is the science of letters and geometry is the science of forms.

In the next stage, everything which is obvious is taken for granted and argument is used only to introduce the unexpected.

The following are some exercise in surveying, which are suitable in the early stage of geometry and mensuration: (1) measuring distance in the playground by means of a tape; finding the length of a pupil's pace; (2) estimating length of roads and fields; verification of the estimates; (3) drawing plans of class-rooms and playing fields and finding their areas.

A good beginning should accomplish certain definite results, whatever method is used to start the course. It should captivate to the individual members of the class; it should provide for review and for the development of basic vocabulary and concepts; it should explain the nature and need of proof; and it should lead into other topics that are to be studied intensively.

Some Special Aspects of Demonstrative Geometry

Definitions: Definition has been defined as a description or explanation of a word, thing or symbol so as to distinguish it from others.

It has also been defined as the explanation of a term by means of others which are more easily understood. It is the designation of the proximate genus and the specific difference. To define a term we must state the proximate genus to which it belongs and the specific difference, i.e., the particulars that distinguish it from the other members of the same genus. The definition of a term should be simpler and more easily understood than the terms or words which are being defined. The functions of a definition are clarity, simplicity and brevity of expression. A definition must be as short as possible and be free from ambiguity. It should not contain anything redundant.

"A parallelogram is a quadrilateral whose opposite sides are equal and parallel" involves a theorem.

Kinds of Definitions

We may divide definitions into several classes:

(1) *Philosophic definitions*: Point, line, plane, space, position, boundary etc. The students are not really concerned with these; all that they require is that they should have a clear concept of what the thing in a formal definition does not help them and does not concern them.

(2) *Explanatory definitions*: Acute angles, alternate angles, complementary angles, diameter of a circle.

(3) Definitions which are essential for logical purpose; parallelogram, isosceles triangle, square, trapezium etc.

The definitions in class (3) should be learnt by heart. It is quite true that in the case of parallelogram, there are many sets, out of which one or two may be taken as definition yet for logical system it is essential to agree on one standard set as fundamental and the pupil must remember what this standard set is.

When should definition be taught?

In the early stages a pupil should not be bothered about the learning of formal definitions. He wants to get a working knowledge of the language of geometry. He must understand the meaning of the words and expressions along with the terms they stand for or represent, rather than be able to define them in set terms. It is sufficient if the pupils understand a 'point' rather than be able to define it.

It we begin our first lessons in geometry with definitions we will be engaged in the jugglery of words which may fail to give a clear picture or the reality to be presented. It is always advisable to start from concrete cases and lead to formulate truths and draw out definitions. It is the knowledge of a thing or its properties which matter and not word picture in set terms which may be quite meaningless for the beginner.

Line: "A line that has the same direction throughout its length" involves the term direction, which is not simpler than straight line. Statement, "A straight line is the shortest distance between two point." is not a definition but a theorem and makes use of distance, a term based upon straight line. There are other definitions which contain flaws. So it is best to accept this term without definition.

If the pupil knows what a straight line is no need to burden his brain with the definition of the line i.e., "A straight line is the shortest distance between two points". "It has no thickness". If we start with definitions we will be giving the pupils memory work without illustrating the real significance of the term. The mastery of definitions is always irksome and a uninteresting task generally disliked by the students. The rigor exercised in memorizing definitions will not add to the intelligence of the pupils.

The mastery of definition should, therefore, be postponed to some later stage when the students know the terms through concrete experience and have had enough practices in handling them in problems. The students sometimes get a wrong notion of the thing through definition which the teacher may find difficult to eradicate later. Some terms are indefinable and any attempt to define them will be futile and will result in confusion. Most of the terms are acceptable without definition i.e., point, line surface etc, From the pedagogic point of view too, definitions are a source of misunderstanding. As far as a the logic that the learning of definitions is supposed to import is concerned, it is most pernicious that definitions are not learnt in a logical way in our schools. It is generally the verbatim repetition of words that is emphasized. The logic will be beneficial only when the pupils themselves arrive at definitions.

But we should not forgot that all definitions do not posses such a value. It is always better that the concept of the term must be clear rather than the definition. This is so because the lack of concept is always a handicap in the progress of learning mathematics. Formal definition should form the end and not the starting point of the study of the term. Emphasis should always be on understanding rather than on memorized statement of definition.

Axioms and postulates

Axioms An axiom is a general mathematical truth accepted without proof, i.e., the whole is greater than the part; if equals be added to or subtracted from equals the results are equal. If equal quantities are multiplied (divided) by equal quantities, the results are equal except that a divisor cannot be zero.

Two quantities which coincide with each other are equal. If two distinct points lie on a plane, then every point of line lies in that plane.

Postulates

Supposition without proof are postulates. In geometry self- evident truths are postulates, e.g.

1. It is possible to draw a line by joining any two points.
2. Two straight lines cannot intersect in more than one point.
3. A terminated straight line in either direction can be produced both ways.

4. A straight line has one mid-point, and only one.
5. An angle can be bisected by one line and only one.
6. It shall be possible to draw a circle with given centre and passing through a given point.
7. All right angles are equal.
8. Any geometric figure can be moved without changing its size or shape.

Axioms should be consistent, free from ambiguity and present no conflict with established knowledge or observe facts. They are like the conventions and rules of a game. We agree to abide by them and play accordingly. They are accepted as true because of their enormity with common experience and sound judgment and they are in no sense 'self-evident' truths. The entire subject of geometry rests upon axioms, postulates and definitions and hence these are frequently called the base of geometry.

Propositions

The propositions in geometry are of two kinds, viz Theorems and Problems.

A theorem is a statement to be proved.

A problem is a geometric construction to be made or a computation to be performed.

Hypothesis and conclusion- Every theorem can be by a complex sentence which has one clause beginning with 'if' and a second clauses beginning with 'then'. The 'if' clause contain the condition which is given or assumed to be true for the sake of argument, and the 'then' clause states what is to be proved. The if clause is called the hypothesis and the then clause is called the conclusion.

Example: If a straight line cuts two parallel lines, then the alternate angles are equal.

There are six parts in the formal proof of any theorem.

1. A statement of the theorem
2. The figure.
3. A statement of what is given(Hypothesis)
4. A statement of what is to be proved (conclusion)
5. Construction if any.
6. An orderly proof. The proof consists of a series of statements of the arguments and the corresponding series of reason. The statement consists of all the items listed under the word 'given' and one or more applications of definitions, axioms, postulates and theorems previously proved.

For each statement of proof there must be a reason. This reason must be one of the following: given, definition, axiom, postulate, theorem previously proved; identity; or conclusion.

Original Exercise

Previously geometrical instruction was confined to the learning of textbook propositions whereas original exercises requiring thought were particularly exclude from the course. Even today the same practice is followed in some of the schools while in other exercises are taught at a later stage. But this is not based on sound principles. This practice seems to be based on a wrong supposition that the beginners cannot think for themselves, till they have mastered a number of set models. Based upon this supposition some text-books do not give any exercises for the first fifty proposition. Some writers even go to the extent of giving printed instructions in the preface that exercises may be omitted

on the first reading. Exercises are considered as something superfluous and unnecessary work. It is thought that this work has no close connection with theorems. Moreover to secure better pass percentage the time which is likely to be spent on exercises should be devoted to the mastery of theorems and book propositions.

This view is erroneous and is based upon lack of mathematical knowledge. The exercises should not be ignored nor their solution postponed to some later stage. The exercises should be solved immediately after the propositions have been learnt.

Nobody advocates a learner to play chees, to study a few chess problems without giving him the chance to play and apply the initial knowledge The study of models will be useful only when the learner gets an opportunity to apply that knowledge and for that reason he must be given a chance to put such lesson in to use. This is true in the case of every human activity, particularly in mathematics where original thinking represents true mathematical work. We learn better by doing. The pupil gets the chance to apply the knowledge, to think, to reason and at the same time recapitulate the knowledge gained. The most appropriate time for solving exercises is immediately after a proposition has been learnt. Knowledge is confirmed through application. We apply the knowledge when it is fresh not because we are likely to forgot, but because we wish that to be fixed.

If exercises are solved immediately after the proposition is learnt, the key to the solution is suggested easily without much effort and extra strain on the memory. Number of exercise can be solved at a time. This gives the pupil a sense of achievement and encourages the student for further work.

If we attempt the exercises after doing fifty or sixty propositions, the reliance becomes a difficult affair and a doubtful matter.

Exercises form a very interesting home-task which creates a spirit of selfhelp and develops independent thought, effort and discovery. The reasons why exercises should be taught at all may be summed up as follows:

1. Only original thinking represents true geometric work. If we concede that it is power and not knowledge that makes the mathematician, and that thinking and not memorizing brings into play the beneficial aspects of mathematical study then the importance of exercise work and the harmfulness of studying a great many readymade proofs must be admitted.
2. Exercises form a much better introduction to do the study of geometry than the study of complex models. It is much easier and interesting to solve a simple and easier exercise than to understand a complicated proof, such as is frequently given on the first pages of a text-book. The solution of exercises gives the beginner a much better idea of the true nature of geometry and guards him against using mechanical modes of study.
3. Exercises arouse the interest of the pupil and prevent him from becoming disgusted with the subject, for every normal youth likes to do things for himself. The discovery of a simple mathematical fact is far more interesting and far more satisfactory than the study of pages of information.
4. Exercises can be much better graded than text-book propositions. It is almost possible to arrange a text-book geometry so that the easiest proofs always come first and the rest follow in order of difficultly. Logical sequence must break the pedagogical sequence. The order of the original riders may, however, be based upon the difficulty of the exercise and the one new element of difficulty should be introduced at a time.

5. Exercises helps in the recapitulation of the mathematical facts previously learnt and sometimes new propositions may be remembered while exercises are solved to familiarize the students with mathematical facts which will enable them to do other complex reasoning. They can be utilized to build up the elements to be used in proving key theorems.
6. The capacity, progress and ability of students in mathematics is to be measured by their ability to solve original exercises and not by their ability to repeat wellknown facts.

The rules of demonstrative geometry

All activity in life is governed by certain rules and regulations. In games, there are rules which every player must learn and observe. To express correctly, one must know the rules of grammar. An individual is expected to follow certain laws and regulations to move properly in society. Similarly there are three basic rules in formal demonstrative geometry.

Rule 1 All geometric statements must be proved deductively. Any conclusion based on measurement or observation even though supported by an experiment should not be accepted as true.

Rule 2 While proving, the steps should be arranged logically i.e., from given data, through other implied facts, and then to the conclusion.

*Rule 3*Each statement in the argument must be supported by one the following:

(a) on assumption if fits the situations,

(b) a known given fact in the theorem.

(c) an accepted definition.

(d) a previously proved geometric statement.

The above rules are essential in order to develop valid proofs.

Correction of Written Work and Exercises

The teacher of mathematics is always anxious to ascertain how far his teaching has gone home and for that purpose he sets exercises and written work. The correction of this work, through a tedium which seems laborious and boring, is yet an indispensable aspect of teacher's work. It must be done in one form or the other.

One way of correcting exercises is to move in the class while the students are at work. Some may have solved the exercise while others may be still busy doing them. The teacher can in this way check the work individually and guide the pupil in their process. Individual mistake should be corrected by the pupils themselves. If many pupils gather at the table of the teacher it may cause disorderliness, so only one pupil should be called at a time.

He can take the uncorrected exercises with him to correct them in his spare time or at home. In this case he should not himself correct the mistakes. The mistakes should be indicated clearly and allowed to be written correctly by the pupils. This gives the pupils an opportunity to understand the nature of his mistakes.

It is probable that there may be some common errors. Such errors should be dealt with the whole class. But these mistakes which are to be dealt by the teacher in the whole class should not be too many, lest they should cause burden. The teacher however must study the mistake carefully and try to locate how and why a mistake has arisen.

Principal mistakes should be explained. Mistakes of computation or process if persisted, should be individually drilled. Mistakes due to carelessness should only be brought to the notice of the pupils. In case a problem is solved wrongly, it is better to set a new problem than to insist on the solution of the same problem.

Correction made by the pupils must be properly scrutinized. Home task whenever set must be properly checked. If the assignment system is in vogue, the teacher should see that the weak students do not copy.

Neatness and accuracy must be insisted on in all written work.

Note on construction

A problem is defined as a geometric construction to be made or as computation to be performed. For example, we may be asked to construct the altitude of a given equilateral triangle or to compute the length of the altitude. The word construct as used in geometry means to draw accurately. The only instruments used in constructions are the straight edge for drawing straight lines and the compasses for drawing arcs and measuring the lengths of line segments.

Parts in the solution of a Construction Problem

(i) A statement of the problem.

(ii) A representation of the given parts.

(iii) A statement of what is given in terms of the drawing.

(iv) A statement of what is to be constructed.

(v) The construction, with a description of and its authority for each step.

(vi) A statement that the required construction has been made.

(vii) A proof of the statement in part (vi).

Discussion of a problem

It is often desirable to discuss the solution of a problem as to the number of solutions, special cases, limitations, and applications.

The statements of construction problems may be used as a reason in proof in the same way as theorems are used.

Induction and Deduction

The technique of making the transition from particular facts to general knowledge about these facts is known as the process of induction. When the pupil measure the three angles of several triangles and finds that in each case the sum of the angles closely approximates to 180 degree or when he cuts out these angles and fits them together and finds that they make a straight angle, he has the background for the 'induction' that the sum of the interior angles of a triangle is equal to two right angles. Such inductions are important in the discovery of truths whether that discoveries made by inductions are in the realm of probable truth can only say, "It is probably true that the sum of the interior angles of a triangle is equal to 180 degrees."

Deduction is the "process of following the network of relations which bind truths together". Deductive reasoning is thus the process of drawing logical inferences from established facts or fundamental assumptions. Demonstrative geometry is primarily a deductive science in which truths stated in the form of theorems can be proved by showing that they are implied by other theorems which have already been proved, definitions that have stated and postulates that have been accepted.

It is evident to the thoughtful student of geometric techniques that the intelligent study of geometry is both inductive and deductive in nature. The teacher should strive to keep before the students the challenge of inductive discovery and the assurance of deductive proof. ■

CHAPTER-18

TOWARDS REFLECTIVE TEACHING

Most teachers develop their classroom skills fairly early in their teaching careers. Teachers entering the profession may find their initial teaching efforts stressful, but with experience they acquire a repertoire of teaching strategies that they draw on throughout their teaching. The particular configuration of strategies a teacher uses constitutes his or her "teaching style". While a teacher's style of teaching provides a means of coping with many of the routine demands of teaching, there is also a danger that it can hinder a teacher's professional growth. How can teachers move beyond the level of automatic or routine responses to classroom situations and achieve a higher level of awareness of how they teach, and of the value and consequences of particular instructional decisions? One way of doing this is through observing and reflecting on one's own teaching, and using observation and reflection as a way of bringing about change. This approach to teaching can be described as "Reflective Teaching".

What is reflection?

Reflection or "critical reflection", refers to an activity or process in which an experience is recalled, considered, and evaluated. It is a response to past experience and involves conscious recall and examination of the experience as a basis for evaluation and decision-making and as a source for planning and action.

How does reflection take place ?

Many different approaches can be employed if one wishes to become a critically reflective teacher, including observation of oneself and others, team teaching, and exploring one's view of teaching through writing. Central to any approach used however is a three part process which involves:

Stage 1 The event itself

The starting point is an actual teaching episode, such as a lesson or other instructional event. While the focus of critical reflection is usually the teacher's own teaching, self-reflection can also be stimulated by observation of another person's teaching.

Stage 2 Recollection of the event

The next stage in reflective examination of an experience is an account of what happened, without explanation or evaluation. Several different procedures are available during the recollection phase, including written descriptions of an event, a video or audio recording of an event, or the use of check lists or coding systems to capture details of the event.

Stage 3 Review and response to the event

Following a focus on objective description of the event, the participant returns to the event and reviews it. The event is now processed at a deeper level, and questions are asked about the experience.

Different approaches to reflection

(i) Peer Observation

Peer observation can provide opportunities for teachers to view each other's teaching in order to expose them to different teaching styles and to provide opportunities for critical reflection on their own teaching.

It involves the following steps

1. Each participant would both observe and be observed

Teachers would work in pairs and take turns observing each other's classes.

2. Pre-observation orientation session

Prior to each observation, the two teachers would meet to discuss the nature of the class to be observed, the kind of material being taught, the teachers' approach to teaching, the kinds of students in the class, typical patterns of interaction and class participation, and any problems that might be expected. The teacher being observed would also assign the observer a goal for the observation and a task to accomplish.

The task would involve collecting information about some aspect of the lesson, but would not include any evaluation of the lesson. Observation procedures or instruments to be used would be agreed upon during this session and a schedule for the observations arranged.

3. The observation

The observer would then visit his or her partner's class and complete the observation using the procedures that both partners had agreed on.

4. Post-observation

The two teachers would meet as soon as possible after the lesson. The observer would report on the information that had been collected and discuss it with the teacher.

(ii) Written accounts of experiences

Another useful way of engaging in the reflective process is through the use of written accounts of experiences.

(iii) Self - Reports

Self -reporting involves completing an inventory or check list in which the teacher indicates which teaching practices were used within a lesson or within a specified time period and how often they were employed. The inventory may be completed individually or in group sessions. The accuracy of self-reports is found to increase when teachers focus on the teaching of specific skills in a particular classroom context and when the self-report instrument is carefully constructed to reflect a wide range of potential teaching practices and behaviours.

Self-reporting allows teachers to make a regular assessment of what they are doing in the classroom. They can check to see to what extent their assumptions about their own teaching are reflected in their actual teaching practice. For example a teacher could use self-reporting to find out the kinds of teaching activities being regularly used, whether all of the programme's goals are being addressed, the degree to which personal goals for a class are being met, and the kind of activities which seem to work well or not to work well.

(iv) Journal Writing

A procedure which is becoming more widely acknowledged as a valuable tool for developing critical reflection is the journal or diary. The goal of journal writing is,

1. to provide a record of the significant learning experiences that have taken place.
2. to help the participant come into touch and keep in touch with the self - development process that is taking place for them.
3. to provide the participants with an opportunity to express, in a personal and dynamic way, their self-development.
4. to foster a creative interaction
   - between the participant and the self-development process that is taking place.
   - between the participant and other participants who are also in the process of self-development.
   - between the participant and the facilitator whose role is to foster such development.

(v) Recording Lessons

For many aspects of teaching, audio or video recording of lessons can also provide a basis for reflection. While there are many useful insights to be gained from diaries and self-reports, they cannot capture the moment to moment processes of teaching. Many things happen simultaneously in a classroom, and some aspects of a lesson cannot be recalled.

At its simplest, a tape recorder is located in a place where it can capture the exchanges which take place during a lesson. With the microphone placed on the teacher's table, much of the teacher's language can be recorded as well as the exchanges of many of the stu-

dents in the class. Where video facilities are available in a school, the teacher can request to have a lesson recorded, or with access to video equipment, students themselves can be assigned this responsibility. A 30 minute recording usually provides more than sufficient data for analysis. The goal is to capture as much of the interaction of the class as possible, both teacher to class and student to student.

Conclusions

A reflective approach to teaching involves changes in the way we usually perceive teaching and our role in the process of teaching. Teachers who explore their own teaching through critical reflection develop changes in attitudes and awareness which they believe can benefit their professional growth as teachers, as well as to improve the kind of support they provide to their students. Reflective teaching suggests that experience alone is insufficient for professional growth, but that experience coupled with reflection can be a powerful impetus for teacher development.■

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